Teachers' Edition

Investigating School Mathematics

Extending the Ideas

Investigating School Mathematics

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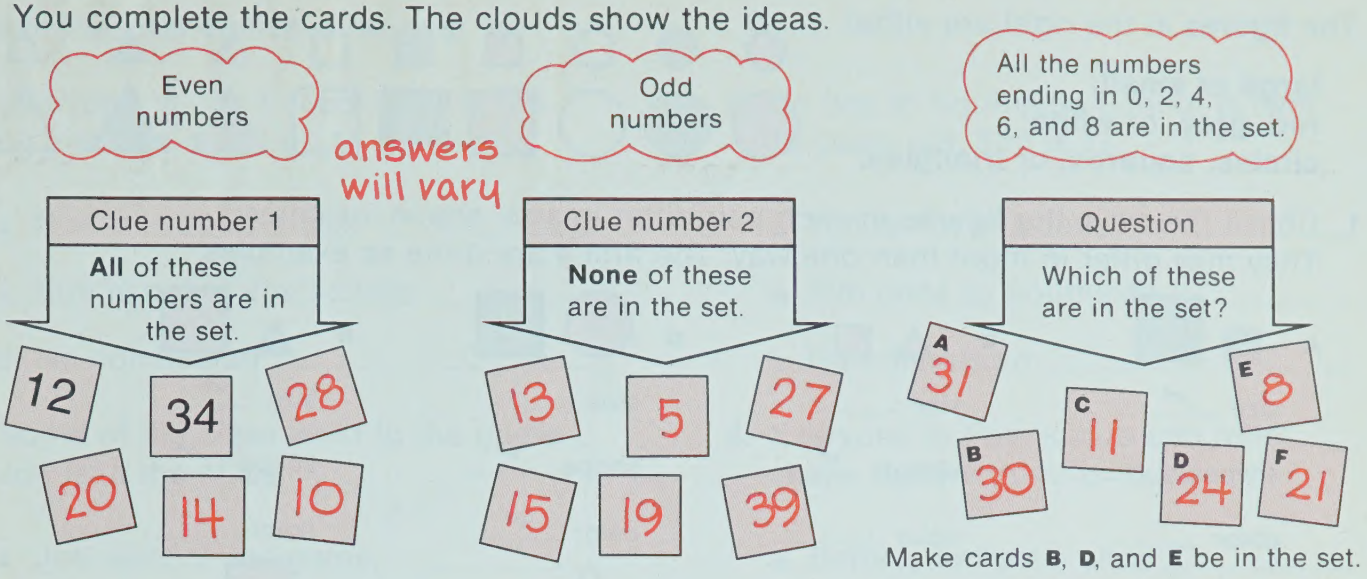
ABCDEFGHIJKL—WC—787654

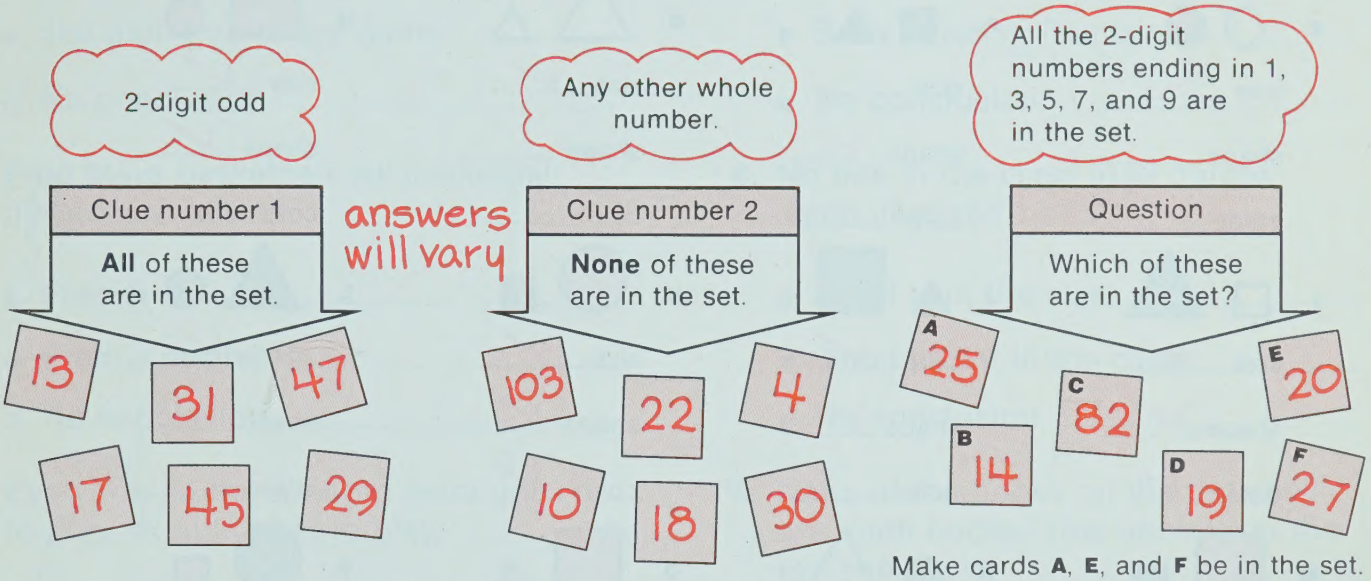
Making Special Sets of Numbers

1. You complete the cards. The clouds show the ideas.



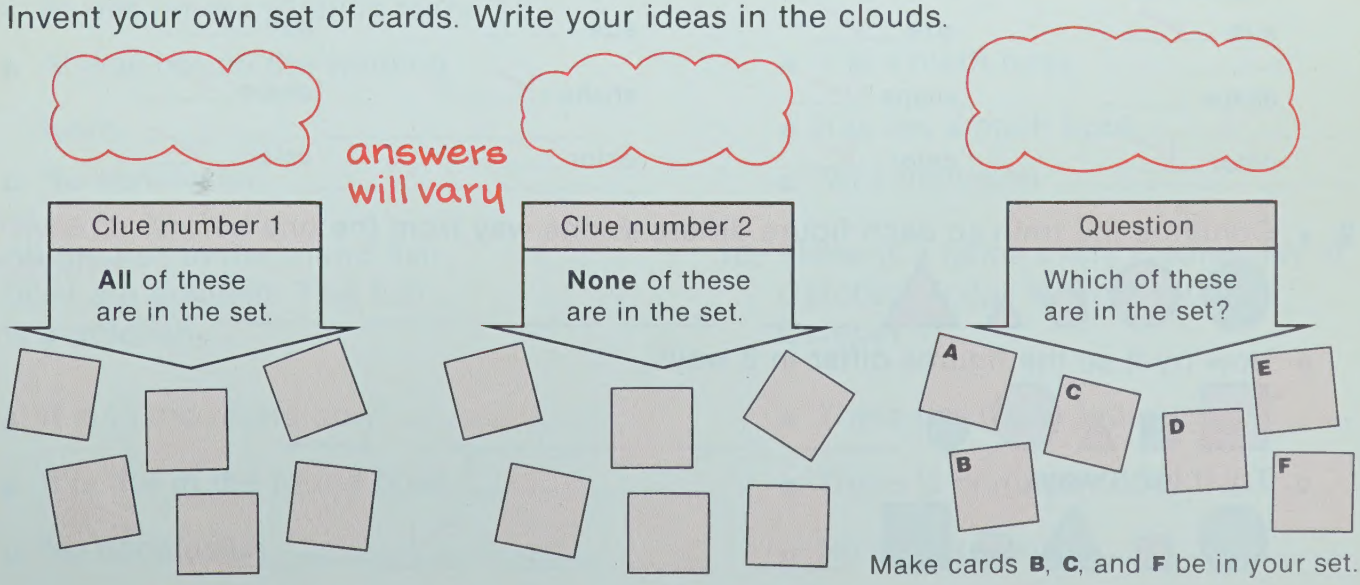
Make cards **B**, **D**, and **E** be in the set.

2.



Make cards **A**, **E**, and **F** be in the set.

3. Invent your own set of cards. Write your ideas in the clouds.



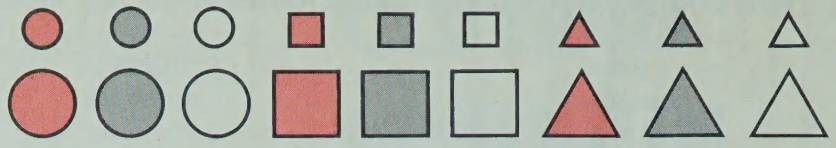
Make cards **B**, **C**, and **F** be in your set.

Exercises 1 and 2 are intended primarily as readiness for having the children create their own set of cards in Exercise 3. Encourage children to come up with new ideas for inventing sets of cards.

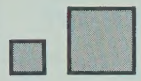
How Do Figures Differ?

The figures at the right are either

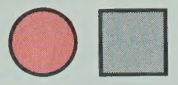
large or **small;**
red, gray, or **white;**
circles, squares, or **triangles.**



1. Check the ways the figures in each pair differ in size, shape, or color. They may differ in more than one way. The first 4 are done as examples.

A 
size ✓
shape
color

B 
size
shape ✓
color

C 
size
shape ✓
color ✓

D 
size ✓
shape ✓
color ✓

E 
size
shape
color ✓

F 
size
shape ✓
color

G 
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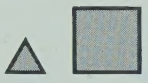
H 
size ✓
shape ✓
color

I 
size ✓
shape ✓
color ✓

J 
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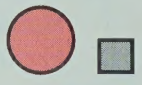
K 
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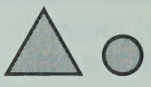
L 
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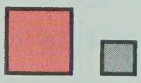
M 
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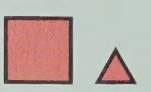
shape
color ✓

N 
size ✓
shape ✓
color ✓

O 
size ✓
shape ✓
color

P 
size ✓
shape ✓
color ✓

2. A Continue the train so each figure differs in one way from the one before. answers will vary

B Now try it so the figures differ in 2 ways.

C Try it for 3 ways.

This lesson can be reinforced by encouraging the children to make cut-outs of the figures described at the top of the page. They may then "see" the various attributes and how figures differ from one another.

How Well Can You Reason?

Check the best choice, A, B, or C.

1. Everyone in Joe's class went to the picnic. Sue didn't go to the picnic.

A Sue is in Joe's class. _____

B Sue is not in Joe's class. ✓

C No conclusion. _____

2. No one in Southside School is over 16 years old. Tim is 17.

A Tim is not in Southside. ✓

B Tim goes to Southside. _____

C No conclusion. _____

3. Some of the class went to the game. Joe is in the class.

A Joe went to the game. _____

B Joe didn't go to the game. _____

C No conclusion. ✓

4. Everyone in Pam's club can roller skate. Betty can roller skate.

A Betty is in Pam's club. _____

B Betty is not in Pam's club. _____

C No conclusion. ✓

5. Fred plays basketball. All basketball players are tall.

A Fred is tall. ✓

B Fred is not tall. _____

C No conclusion. _____

6. No one in the class likes purple. Janet likes red.

A Janet is in the class. _____

B Janet in not in the class. _____

C No conclusion. ✓

7. Everyone on the winning team got to play. Al didn't get to play.

A Al was on the winning team. _____

B Al was not on the winning team. _____

C No conclusion. ✓

8. None of the books on the top shelf are math books. This book is on the top shelf.

A It is a math book. _____

B It is not a math book. ✓

C No conclusion. _____

9. All the fish in the round fish bowl are goldfish. This fish is a goldfish.

A It is in the round bowl. _____

B It is not in the round bowl. _____

C No conclusion. ✓

10. There is a game every Wednesday in October. Today is Wednesday, October 17.

A There is a game today. ✓

B There is no game today. _____

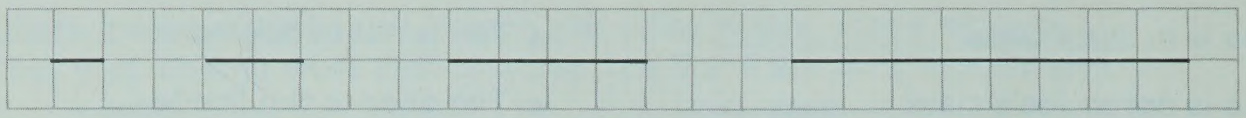
C No conclusion. _____

Some important benefits can be gained from this lesson by utilizing a discussion after the completion of the page.

Patterns in Units

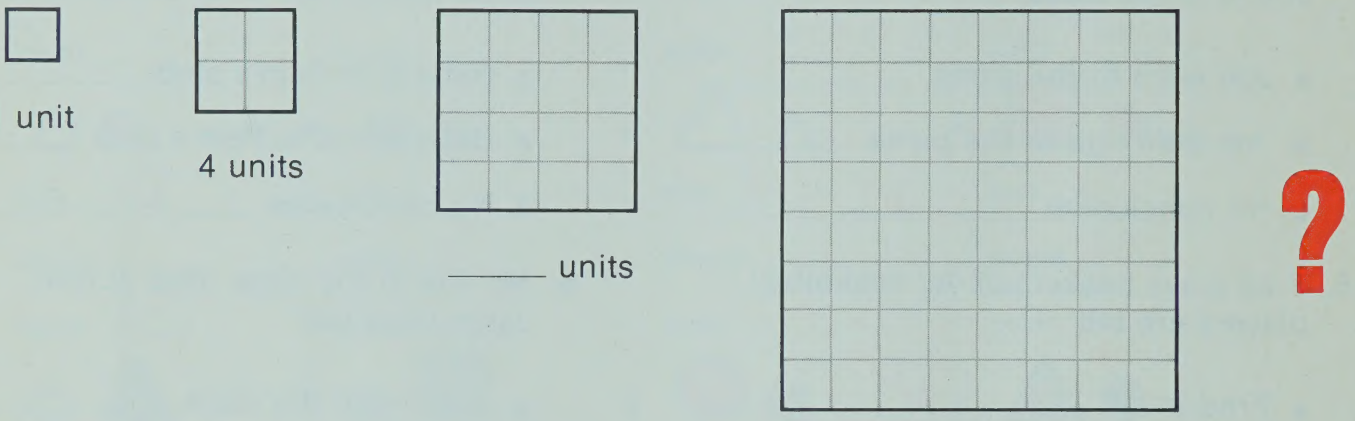
Study the pattern. Then give the missing number of units, including the number of units in the next figure in the pattern.

1.



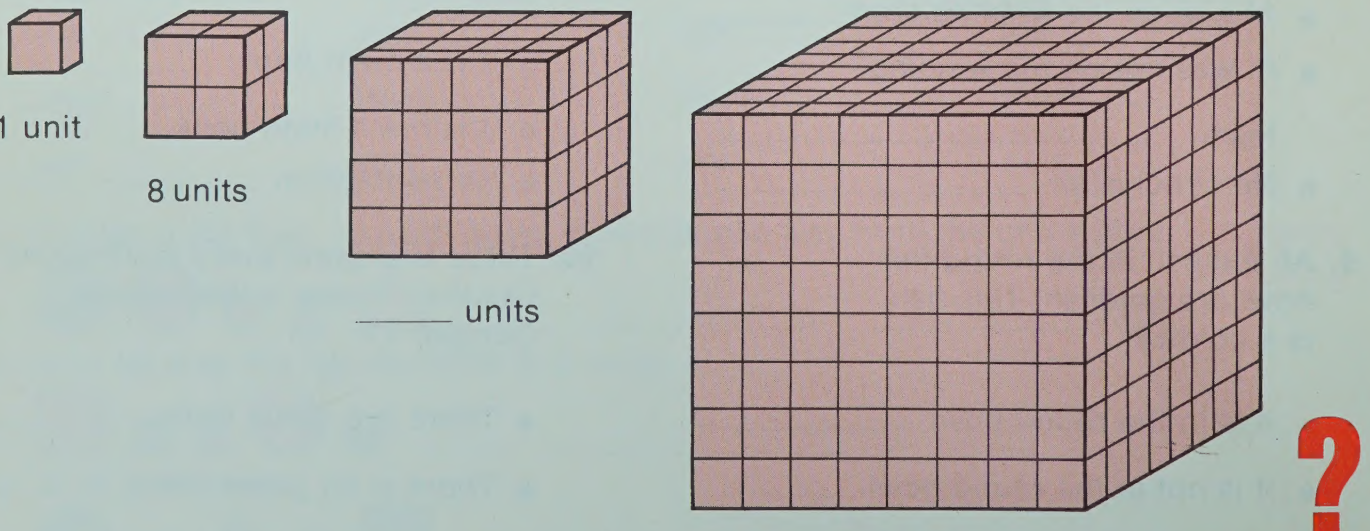
1 unit 2 units 4 units _____ units _____ units

2.



1 unit 4 units _____ units _____ units _____ units

3.



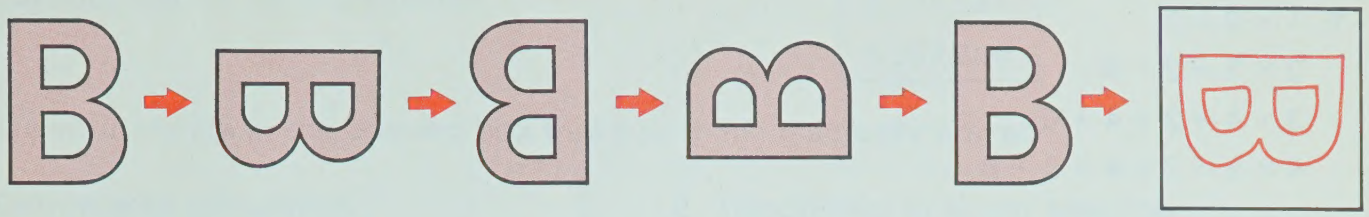
1 unit 8 units _____ units _____ units _____ units

This lesson illustrates the effect that doubling the dimensions of a given figure has on length, area, and volume.

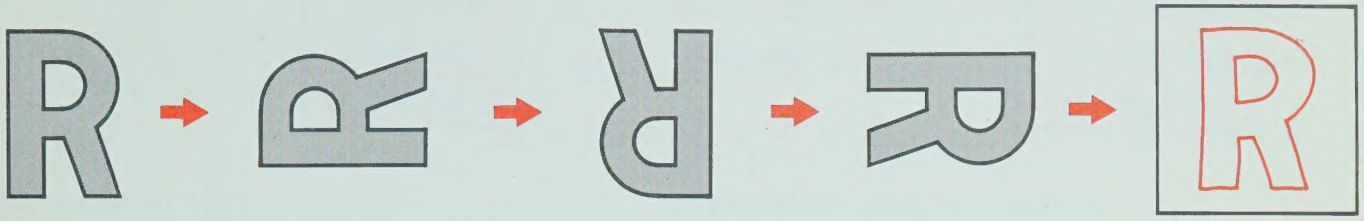
●*Figure "Sequences"*

Can you draw the next figure in each figure "sequence?"

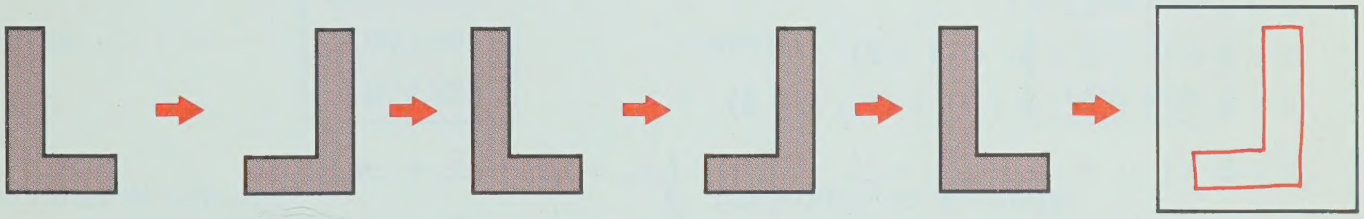
1. next figure



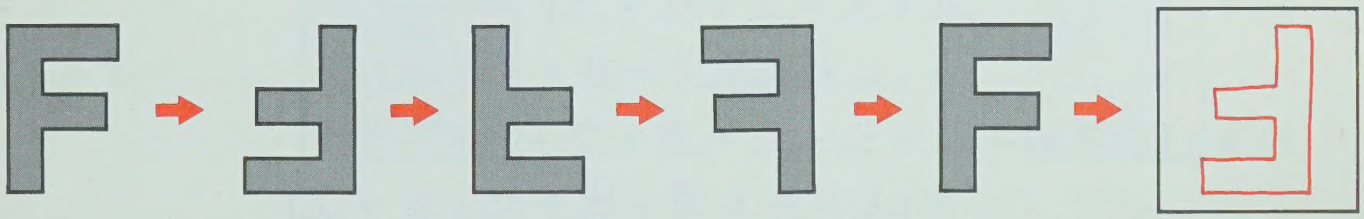
2.



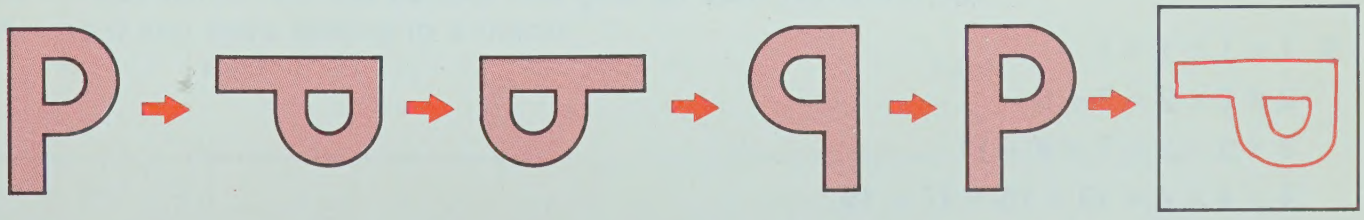
3.



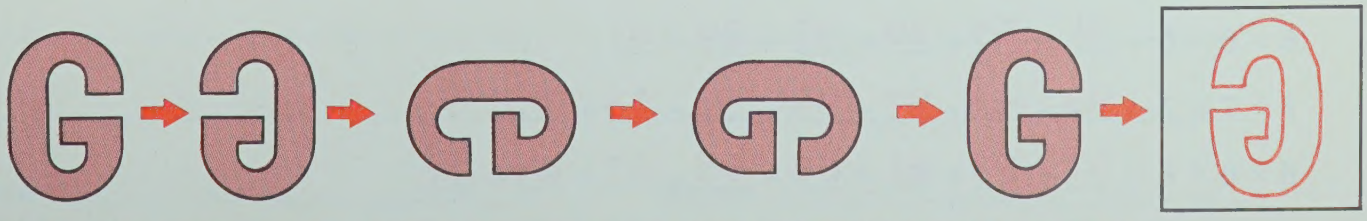
4.



5.



6.



It might be helpful to encourage children to describe the type of motion involved in each step of a given sequence.

Number Patterns

Study the pattern. Give the missing addends and check your result by finding the product and sum.

1. $1 \times 2 = 2$

$2 \times 3 = 2 + 4$

$3 \times 4 = 2 + 4 + 6$

$4 \times 5 = 2 + 4 + 6 + 8$

$5 \times 6 = 2 + 4 + 6 + 8 + 10$

$6 \times 7 = 2 + 4 + 6 + 8 + 10 + 12$

$7 \times 8 = 2 + 4 + 6 + 8 + 10 + 12 + 14$

$8 \times 9 = 2 + 4 + 6 + 8 + 10 + 12 + 14 + 16$

2. $1 \times 1 = 1$

$2 \times 2 = (1 + 2) + 1$

$3 \times 3 = (1 + 2 + 3) + (1 + 2)$

$4 \times 4 = (1 + 2 + 3 + 4) + (1 + 2 + 3)$

$5 \times 5 = (1 + 2 + 3 + 4 + 5) + (1 + 2 + 3 + 4)$

$6 \times 6 = (1 + 2 + 3 + 4 + 5 + 6) + (1 + 2 + 3 + 4 + 5)$

$7 \times 7 = (1 + 2 + 3 + 4 + 5 + 6 + 7) + (1 + 2 + 3 + 4 + 5 + 6)$

$8 \times 8 = (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) + (1 + 2 + 3 + 4 + 5 + 6 + 7)$

3. $1 \times 1 \times 1 = 1$

$2 \times 2 \times 2 = 3 + 5$

$3 \times 3 \times 3 = 7 + 9 + 11$

$4 \times 4 \times 4 = 13 + 15 + 17 + 19$

$5 \times 5 \times 5 = 21 + 23 + 25 + 27 + 29$

$6 \times 6 \times 6 = 31 + 33 + 35 + 37 + 39 + 41$

$7 \times 7 \times 7 = 43 + 45 + 47 + 49 + 51 + 53 + 55$

$8 \times 8 \times 8 = 57 + 59 + 61 + 63 + 65 + 67 + 69 + 71$

Liz said, "The population of the Chicago area is about → 7,000,000."

One of these numbers was the actual population.
Circle your choice for this number. →

7,724,947
6,978,947
6,287,569

Circle your choice in the colored box that you think is actually correct.

1. Detroit area population
about → 4,000,000

actual →

4,623,578
3,349,867
4,199,931

2. Telephones in North America
about → 130,000,000

actual →

129,518,103
120,587,011
146,867,000

3. Car sales in 1971
about → 8,600,000

actual →

8,584,592
7,693,667
8,974,523

4. Montreal area population
about → 2,700,000

actual →

2,743,203
2,190,223
3,697,000

5. Number of minutes in a year
about → 500,000

actual →

531,634
525,600
448,239

6. Take-offs and landings at busiest airport
about → 650,000

actual →

689,035
641,429
695,000

Look up some large number facts of your own and report them in the spaces below. On the bottom line give an approximation you might use if you were talking to a friend.

Fact

_____

_____

_____

_____

About _____

Fact

_____

_____

_____

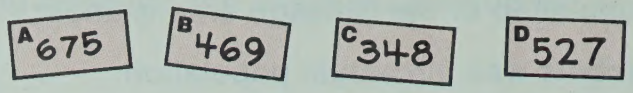
_____

About _____

This lesson is intended to help children think about what approximations actually mean when used in everyday language. An important aspect of the lesson is found at the bottom of the page where children find facts that have meaning to them.

Large Numbers

Make 4 slips of paper like these.



You can "write" different numbers with your slips of paper.

EXAMPLES:

1. Using 3 slips at a time, how many numbers can you show and record if the C slip always tells the number of millions?

EXAMPLE

348,469,527

348,469,675

348,527,469

348,527,675

348,675,469

348,675,527

2. Using 4 slips at a time, how many numbers can you record if the B slip shows the number of millions?

EXAMPLE

675,469,348,527

675,469,527,348

348,469,527,675

348,469,675,527

527,469,348,675

527,469,675,348

3. Show each of these.

A Largest with 3 slips

675,527,469

B Smallest with 2 slips

348,469

C Smallest with all slips

348,469,527,675

D Smallest with 3 slips

348,469,527

E A—millions, C—billions and D—thousands

348,675,527,469

F B—thousands, D—millions and A—billions

675,527,469,348

4. Complete these inequalities using 2 slips on each side.

EXAMPLE

675,348 > 469,527

Answers will vary

_____ > _____

_____ < _____

_____ < _____

_____ > _____

_____ > _____

_____ < _____

_____ < _____

It is possible for the children to complete this page without making slips of paper. However, much will be gained from the lesson if they actually make the slips of paper and use them as they complete the exercises.

Counting in Base Four

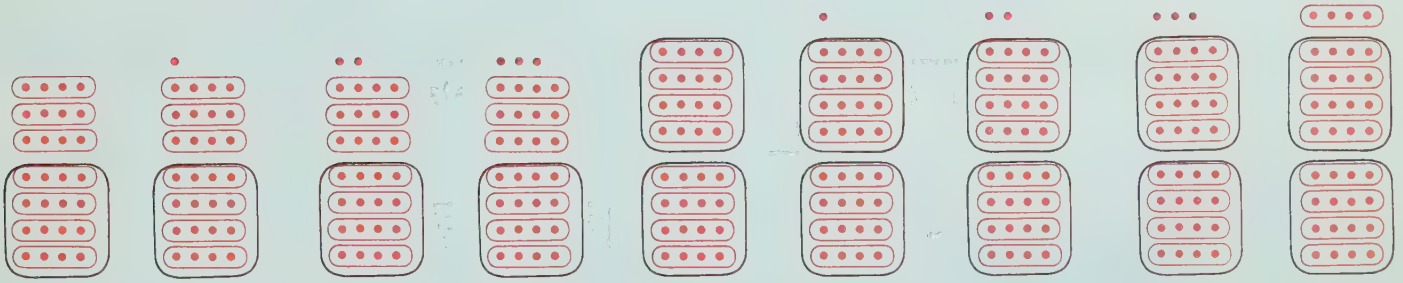
1. Study the counting sequence below to see how any number of dots can be written using only the digits 0, 1, 2, and 3. Give the missing numerals. For these exercises you may omit the base-four notation beside the numerals.

For example, we will write
32 for $32_{(4)}$

0 1 2 3 10 11 12 13 20 21

22 23 30 31 32 33 100 101 102

103 110 111 112 113 120 121 122 123



130 131 132 133 200 201 202 203 210

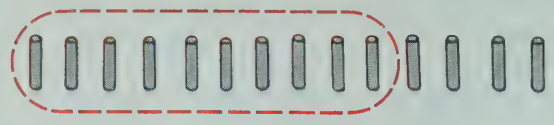
2. Do you see the counting pattern? Can you count farther without the sets of dots?

211	212	213	220	221	222	223	230	231
232	233	300	301	302	303	310	311	312
313	320	321	322	323	330	331	332	333
1000	1001	1002	1003	1010	1011	1012	1013	1020
1021	1022	1023	1030	1031	1032	1033	1100	1101

Help the children see how counting the digits 0, 1, 2, 3, relates to counting with the digits 0 through 9 in the base ten system. The pattern involved is the same for each system.

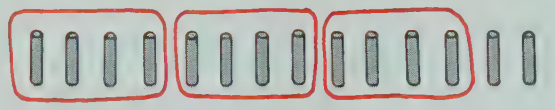
Other Bases for Numerals

1. Draw rings to group the sticks as indicated. Then fill in the blanks.

A In base ten group by **tens:** 

1 tens and 4

We write 14

B In base four group by **fours:** 

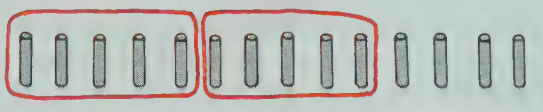
3 fours and 2

We write $32_{(4)}$

C In base six group by **sixes:** 

2 sixes and 2

We write $22_{(6)}$

D In base five group by **fives:** 

2 fives and 4

We write $24_{(5)}$

E In base eight group by **eights:** 

1 eights and 6.

We write $16_{(8)}$

2. Think of each sack as containing marbles. The number of marbles is given in a base other than ten. Give the base ten numeral for each one.

A $23_{(4)}$ ⋮⋮

B $22_{(4)}$ 10

C $33_{(5)}$ 18

D $42_{(6)}$ 26

E $51_{(8)}$ 41

F $30_{(6)}$ 18

G $77_{(8)}$ 63

H $40_{(5)}$ 20

I $123_{(4)}$ 27

J $312_{(4)}$ 54

K $453_{(6)}$ 177

L $537_{(8)}$ 351

In giving answers for exercise 2 the children need to think about the grouping involved in the various bases. Extra help may be required for parts **I, J, K,** and **L.**

Computing in Another Base

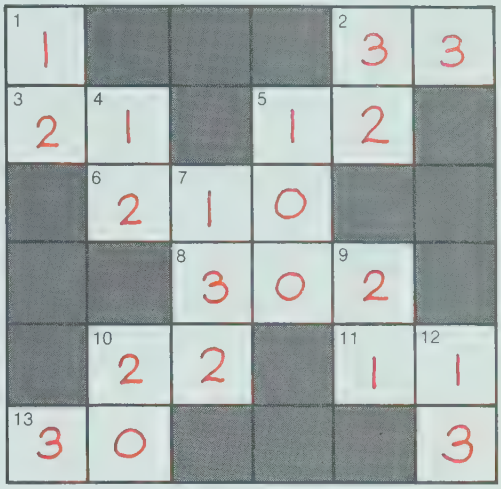
1. This is a base 4 puzzle. Give all your answers in base 4 without the $_{(4)}$.
For example, write 32 instead of $32_{(4)}$.

Across

2. largest 2-digit numeral
3. $3_{(4)} \times 3_{(4)}$
5. 1 four and 2
6. thirty-six
8. half a hundred
10. ten
11. a nickel is ? cents
13. noon

Down

1. $3_{(4)} + 3_{(4)}$
2. $2_{(4)} \times 13_{(4)}$
4. half a dozen
5. 4 fours
7. $3_{(4)} \times 22_{(4)}$
9. area: 
10. 
12. $2_{(4)} + 3_{(4)} + 2_{(4)}$



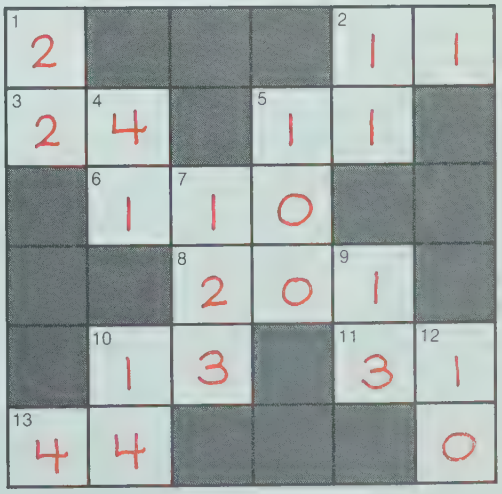
2. This is a base 5 puzzle. Give all your answers in base 5 without the $_{(5)}$.
For example, write 24 instead of $24_{(5)}$.

Across

2. $3_{(5)} + 3_{(5)}$
3. 2 sevens
5. half dozen
6. thirty
8. fifty-one
10. $2_{(5)} \times 2_{(5)} \times 2_{(5)}$
11. $4_{(5)} \times 4_{(5)}$
13. largest 2-digit numeral

Down

1. dozen
2. $2_{(5)} \times 3_{(5)}$
4. 3 sevens
5. area: 
7. thirty-eight
9. $4_{(5)} + 4_{(5)}$
10. 0, 1, 4, ?, 16, 25, . . .
12. 



Using Different Symbols

Suppose you used some different symbols for base 4 numerals. Try the counting and computing using the new symbols in the box.

$0_{(4)} = a$

$1_{(4)} = b$

$2_{(4)} = c$

$3_{(4)} = d$

1. Complete the counting. Hint: Your work on page 9 should help you.

a, b, c, d, ba, bb, bc, bd, ca, cb, cc, cd, da,

db, dc, dd, baa, bab, bac, bad.

2. Complete each table.

+	a	b	c	d
a	a	b	c	d
b	b	c	d	ba
c	c	d	ba	bb
d	d	ba	bb	bc

×	a	b	c	d
a	a	a	a	a
b	a	b	c	d
c	a	c	ba	bc
d	a	d	bc	cb

3. Solve.

A b c + b = bd

B b c + c = ca

C b c + d = cb

D d c + b = dd

E d c + c = baa

F d c + d = bab

G b c × b = bc

H b c × c = da

I b c × d = bac

J c a × b = ca

K c a × c = baa

L c a × d = bca

4. Make up some symbols of your own for base 5. Try some counting and computing. Answers will vary

$0_{(5)} =$ _____

$1_{(5)} =$ _____

$2_{(5)} =$ _____

$3_{(5)} =$ _____

$4_{(5)} =$ _____

If children have difficulty with these new symbols for numbers, remind them that these new symbols are just another way of expressing base-four numbers.

The number 2 can be written as a sum in just one way.
We won't use 0 as an addend on this page.

$2 = 1 + 1$

The number 3 can be written as a sum in 3 ways.

$3 = 2 + 1$
$3 = 1 + 2$
$3 = 1 + 1 + 1$

The number 4 can be written as a sum in 7 ways.

$4 = 3 + 1$
$4 = 1 + 3$
$4 = 2 + 2$
$4 = 2 + 1 + 1$
$4 = 1 + 2 + 1$
$4 = 1 + 1 + 2$
$4 = 1 + 1 + 1 + 1$

1. How many ways can you write 5 as a sum?
Some examples are given to get you started.

2 addends	3 addends	4 addends	5 addends
$5 = 4 + 1$	$5 = 3 + 1 + 1$	$5 = 2 + 1 + 1 + 1$	$5 = 1 + 1 + 1 + 1 + 1$
$5 = 1 + 4$	$5 = 1 + 3 + 1$	$5 = 1 + 1 + 1 + 2$	
$5 = 3 + 2$	$5 = 1 + 1 + 3$	$5 = 1 + 2 + 1 + 1$	
$5 = 2 + 3$	$5 = 2 + 2 + 1$	$5 = 1 + 1 + 2 + 1$	
	$5 = 2 + 1 + 2$		
	$5 = 1 + 2 + 2$		

Keep trying. There are 15 ways altogether.

2. How many ways can you find to write 6 as a sum? Organize your work carefully—there are 31.

2 addends	3 addends	4 addends	5 addends	6 addends
$6 = 1 + 5$	$6 = 3 + 2 + 1$	$6 = 2 + 2 + 1 + 1$	$6 = 2 + 1 + 1 + 1 + 1$	$6 = 1 + 1 + 1 + 1 + 1 + 1$
$6 = 5 + 1$	$6 = 3 + 1 + 2$	$6 = 2 + 1 + 2 + 1$	$6 = 1 + 2 + 1 + 1 + 1$	
$6 = 4 + 2$	$6 = 2 + 3 + 1$	$6 = 2 + 1 + 1 + 2$	$6 = 1 + 1 + 2 + 1 + 1$	
$6 = 2 + 4$	$6 = 2 + 1 + 3$	$6 = 1 + 2 + 1 + 2$	$6 = 1 + 1 + 1 + 2 + 1$	
$6 = 3 + 3$	$6 = 1 + 2 + 3$	$6 = 1 + 2 + 2 + 1$	$6 = 1 + 1 + 1 + 1 + 2$	
	$6 = 1 + 3 + 2$	$6 = 1 + 1 + 2 + 2$		
	$6 = 4 + 1 + 1$	$6 = 3 + 1 + 1 + 1$		
	$6 = 1 + 1 + 4$	$6 = 1 + 3 + 1 + 1$		
	$6 = 1 + 4 + 1$	$6 = 1 + 1 + 3 + 1$		
	$6 = 2 + 2 + 2$	$6 = 1 + 1 + 1 + 3$		

Note: Would you believe 7 has 63 ways, 8 has 127 ways, and 9 has 255 ways?

The note at the bottom of the page is to encourage those children who are interested to organize their work so they can observe all the ways to write these sums.

Repeated Subtraction

You can "get to zero" by subtracting ones, twos, threes, fours, sixes, or twelve from 12. You can't get to zero by subtracting fives. Try it in the space below.

12	12	12	12	12
− 2	− 3	− 4	− 6	−12
10	9	8	6	0
− 2	− 3	− 4	− 6	
8	6	4	0	
− 2	− 3	− 4		
6	3	0		
− 2	− 3			
4	0			
− 2				
2				
− 2				
0				

For each number below, how many different numbers can you find that will "get you to zero" by repeated subtraction.

30 1, 2, 3, 5, 6, 10, 15, 30

42 1, 2, 3, 6, 7, 14, 21, 42

60 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

144 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

Through a process of repeated subtraction children are finding factors of these numbers.

Solving "Tricky" Equations

1. Try to solve these equations without doing any pencil-paper computing.

A $(6 \times 4) \div 4 = n$

$n =$ 6

B $(9 \times 8) \div 8 = n$

$n =$ 9

C $(48 \div 8) \times 8 = n$

$n =$ 48

D $(63 \div 9) \times 9 = n$

$n =$ 63

E $(17 \times 6) \div 6 = n$

$n =$ 17

F $(38 \times 9) \div 9 = n$

$n =$ 38

G $(322 \div 7) \times 7 = n$

$n =$ 322

H $(0 \div 13) \times 13 = n$

$n =$ 0

I $(483 \times 29) \div 29 = n$

$n =$ 483

2. Now try these, but be careful.

A $(12 \times 4) \div 4 = n$

$n =$ 12

B $(12 \times 4) \div 8 = n$

$n =$ 6

C $(12 \times 4) \div 2 = n$

$n =$ 24

D $(24 \times 6) \div 3 = n$

$n =$ 48

E $(24 \times 6) \div 12 = n$

$n =$ 12

F $(24 \times 3) \div 6 = n$

$n =$ 12

G $(32 \div 4) \times 8 = n$

$n =$ 64

H $(32 \div 8) \times 4 = n$

$n =$ 16

I $(32 \times 4) \div 8 = n$

$n =$ 16

3. Are you ready? Now try these.

A $(6 \times 3) \div n = 6$

$n =$ 3

B $(12 \times 4) \div n = 12$

$n =$ 4

C $(23 \times n) \div 5 = 23$

$n =$ 5

D $(47 \times n) \div 8 = 47$

$n =$ 8

E $(8 \times 4) \div n = 8$

$n =$ 4

F $(8 \times 4) \div n = 16$

$n =$ 2

G $(8 \times 4) \div n = 4$

$n =$ 8

H $(8 \times 4) \div n = 32$

$n =$ 1

I $(24 \div n) \times 6 = 24$

$n =$ 6

J $(24 \div n) \times 6 = 48$

$n =$ 3

K $(24 \div n) \times 6 = 12$

$n =$ 12

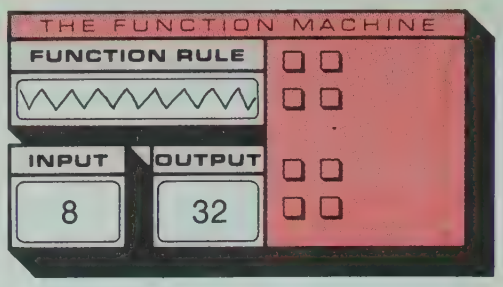
L $(24 \div n) \times 6 = 72$

$n =$ 2

In exercise 2 help the children see the relationships between various sets of exercises. Note that in part **B** the answer is half of part **A** and that in part **C** the answer is double part **A**.

A "Broken" Function Machine

Since the rule screen is out of order on this function machine, you do not have to give a rule.
Show you know each rule by giving the missing numbers in the tables below. Each table has a different rule.



7×n

1. Function Rule

Input	Output
7	49
6	42
9	63
5	35
8	56
4	28

Output is last digit

2. Function Rule

Input	Output
12	2
38	8
543	3
627	7
487	7
56	6

Input even, output is 0
Input odd, output is 5.

3. Function Rule

Input	Output
4	0
3	5
6	0
5	5
7	5
8	0

n×2 output is last digit

4. Function Rule

Input	Output
3	6
4	8
5	0
6	2
7	4
8	6

n×3 output is last digit

5. Function Rule

Input	Output
2	6
3	9
4	2
5	5
6	8
7	1

n×4 output is last digit

6. Function Rule

Input	Output
1	4
2	8
3	2
4	6
5	0
6	4

n×6 output is last digit

7. Function Rule

Input	Output
5	0
3	8
6	6
2	2
7	2
4	4

n+output = 10

8. Function Rule

Input	Output
2	8
8	2
3	7
5	5
6	4
4	6

Often children are able to work out functions without being able to verbalize what they have found. However, if children want to verbalize these function rules encourage them to do so.

Finding an Unknown Number

1. In each part, each card covers the same number of discs.
Write the hidden number of discs on the card.

A 

23 discs in all

B 

24 discs in all

C 

36 discs in all

D 

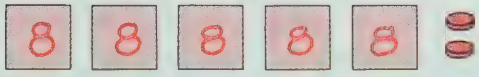
32 discs in all

E 

30 discs in all

F 

43 discs in all

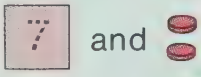
G 

42 discs in all

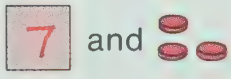
H 

40 discs in all

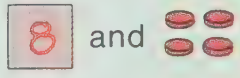
2. In these exercises, 3 ☐ will mean the same as showing 3 cards. For example, part A above would be shown as 4 ☐ and ●●. Again write the unknown number on the card.

A 3  and ●

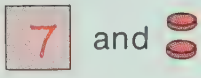
23 in all

B 6  and ●●

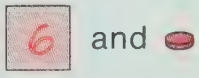
45 in all

C 5  and ●●

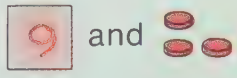
44 in all

D 7  and ●●

51 in all

E 9  and ●

55 in all

F 8  and ●●

75 in all

3. The example below shows a new way to write the examples above.

4 ☐ and ●●● ⟶ 4 ☐ + 5 = 29

29 in all

Write the unknown number on the box.

A 6  + 3 = 33

B 7  + 6 = 48

C 4  + 8 = 44

D 5  + 8 = 58

E 9  + 5 = 68

F 8  + 6 = 70

G 3  + 7 = 40

H 7  + 4 = 60

I 5  + 9 = 49

J 9  + 7 = 61

K 6  + 12 = 30

L 8  + 15 = 31

If children have difficulty understanding the ideas on this page, have them use 3 x 5 cards to cover up a given number of counters and "visualize" the concepts involved.

Solving Equations

On this page, letters will be used in place of cards.

$4\boxed{5} + 6 = 26 \longrightarrow 4n + 6 = 26$

$n = 5$

Also you can think of $4n - 6$ as meaning 6 discs less than those under the cards.

1. Solve the equations.

A $6x + 5 = 29$	F $3m - 4 = 26$	K $6t + 1 = 19$	P $8p - 12 = 20$
$x =$ 4	$m =$ 10	$t =$ 3	$p =$ 4
B $5t - 3 = 32$	G $9a - 1 = 80$	L $8d - 2 = 38$	Q $7t + 25 = 32$
$t =$ 7	$a =$ 9	$d =$ 5	$t =$ 1
C $4n + 1 = 33$	H $5n + 4 = 39$	M $6a - 6 = 6$	R $9b + 7 = 7$
$n =$ 8	$n =$ 7	$a =$ 2	$b =$ 0
D $5b - 2 = 28$	I $8s + 10 = 82$	N $7c + 8 = 29$	S $10c + 4 = 34$
$b =$ 6	$s =$ 9	$c =$ 3	$c =$ 3
E $9r + 5 = 50$	J $7y - 10 = 18$	O $5n - 5 = 0$	T $12m - 8 = 40$
$r =$ 5	$y =$ 4	$n =$ 1	$m =$ 4

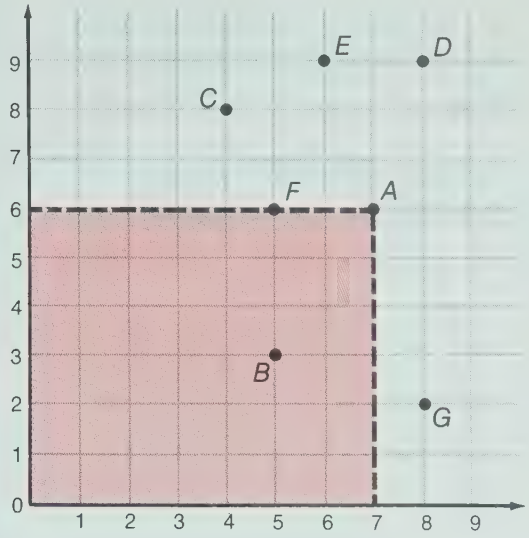
2. Solve the equations.

A $2n = n + 6$	E $3n = n + 8$	I $6x = 3x + 12$	M $2n + 3 = n + 5$
$n =$ 6	$n =$ 4	$x =$ 4	$n =$ 2
B $2r = r + 8$	F $4a = 2a + 12$	J $5y = y + 20$	N $3r + 2 = r + 8$
$r =$ 8	$a =$ 6	$y =$ 5	$r =$ 3
C $3s = 2s + 5$	G $5c = 3c + 10$	K $7m = m + 12$	O $5y + 1 = 2y + 13$
$s =$ 5	$c =$ 5	$m =$ 2	$y =$ 4
D $5t = 4t + 7$	H $7d = 5d + 4$	L $8s = 3s + 15$	P $8g + 3 = 3g + 28$
$t =$ 7	$d =$ 2	$s =$ 3	$g =$ 5

Do not try to teach rules of algebra before solving these equations. Rather, help the children relate these exercises to their work with cards and counters on the previous page.

Area and Graphs

1. You can think of point *A* as one corner of a rectangular region. Using the units of the graph, what is the area of the region? 42



2. What is the area of the rectangle having one corner as point

B? 15 *E*? 54

C? 32 *F*? 30

D? 72 *G*? 16

3. What is the area of the rectangle given by each of these points?

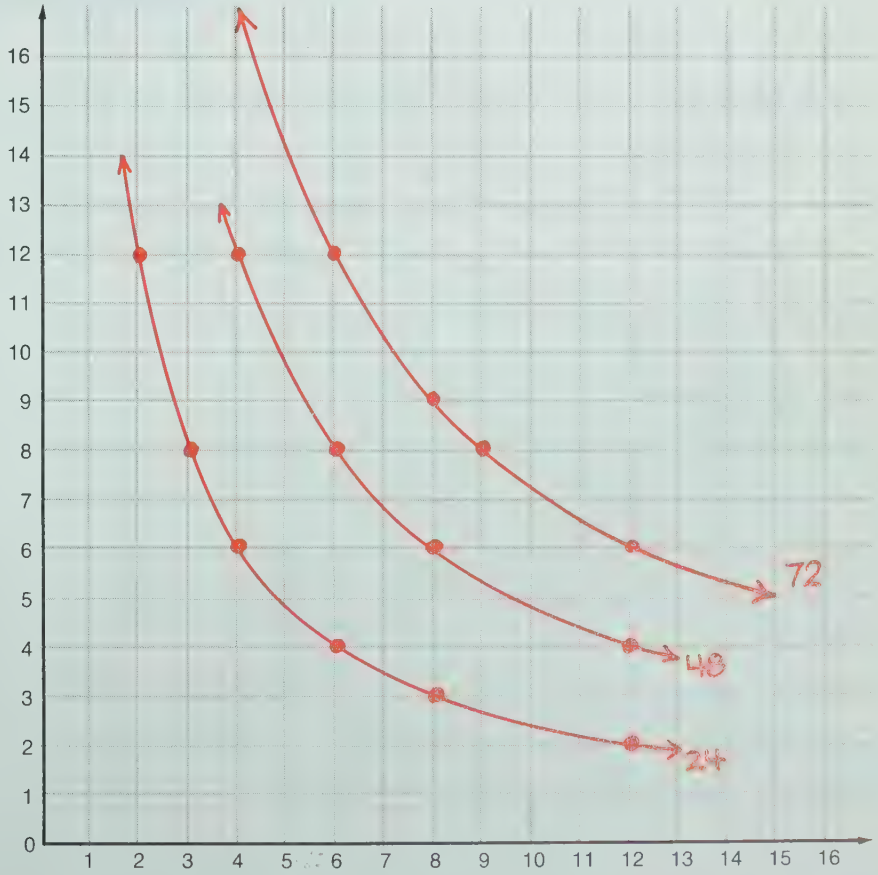
(5, 8) 40 (6, 7) 42 (8, 5) 40 (6, 6) 36

(9, 7) 63 (9, 8) 72 (7, 8) 56 (9, 9) 81

4. How many points can you find for rectangles that have area 24? Connect your points with a "smooth curve."

5. Repeat Exercise 4 for Area 48.

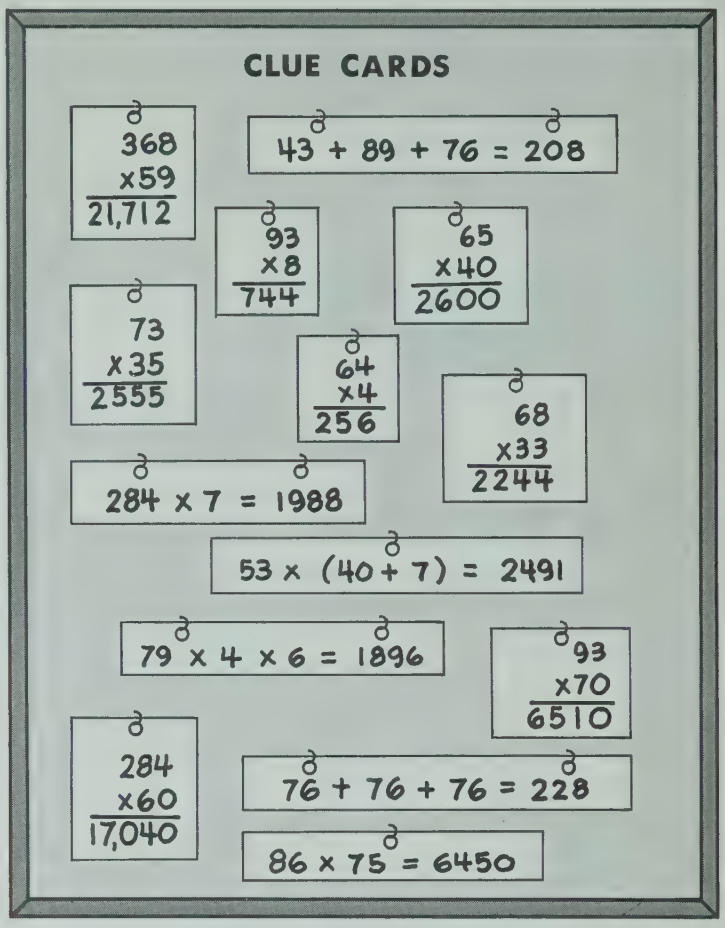
6. On a separate sheet of graph paper, try this for area 72.



Using the Basic Principles

Using the clue cards, you can solve the problems below with very little work. Try them.

Remember, if you do much work, you may not be taking advantage of the cards.



1. 284 × 67 = 19,028

2. 79 × 24 = 1896

3. 89 + 43 + 76 = 208

4. 86 × 75 = 6450

5. 367 × 59 = 21,653

6. 284 × 61 = 17,324

7. $75 \times 86 = n$

$n =$ 6450

8. $24 \times 79 = n$

$n =$ 1896

9. $368 \times 59 = n$

$n =$ 21,712

10. $73 \times (30 + 5) = n$

$n =$ 2555

11. $64 + 64 + 64 + 65 = n$

$n =$ 257

12. $47 \times (50 + 3) = n$

$n =$ 2491

13. 65 × 44 = 2860

14. 93 × 78 = 7254

15. 76 × 3 = 228

16. $64 \times 8 = n$

$n =$ 512

17. $32 \times 68 = n$

$n =$ 2176

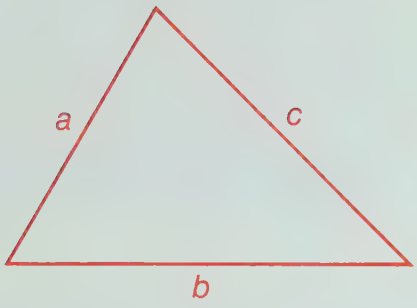
18. $35 \times (70 + 3) = n$

$n =$ 2555

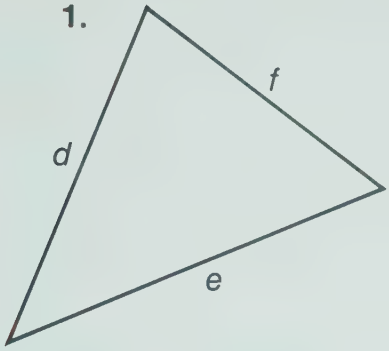
Encourage the children to use the clue cards as much as possible. For example, in exercise 1, the children should observe that 67 × 284 is the same as (60 × 284) + (7 × 284). In exercise 11, they should observe that the answer is (64 × 4) + 1.

Congruent Segments

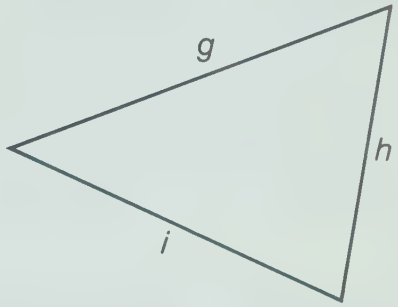
Each triangle has one side that is congruent to *a*, one side congruent to *b*, and one side congruent to *c*. See if you can find and record them below.



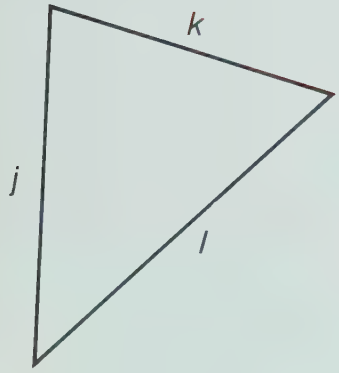
1.



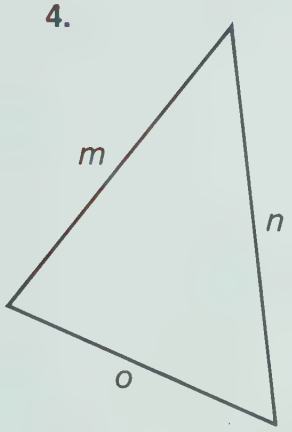
2.



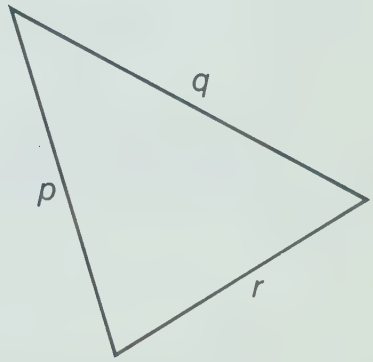
3.



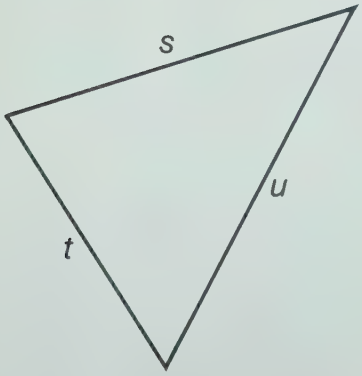
4.



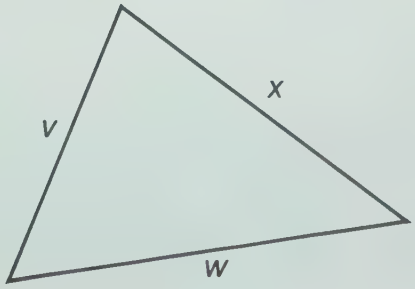
5.



6.



7.



The segments congruent to *a* f, h, k, o, r, t, v

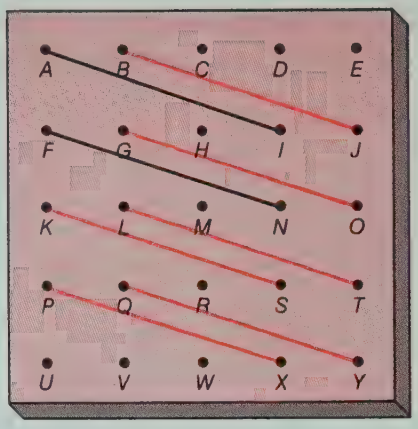
The segments congruent to *b* e, g, l, n, q, u, w

The segments congruent to *c* d, i, j, m, p, s, x

If children have difficulty with this exercise have them cut out a triangle that is congruent to the given triangle. Then by labeling the sides a, b, and c, and placing the triangle on the other triangles, they should be able to find and record the congruent segments.

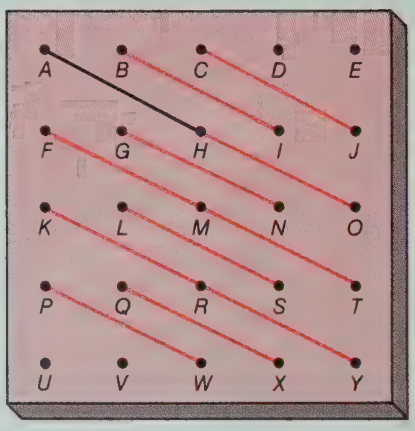
Parallel and Perpendicular Segments

1. Segment *FN* is parallel to segment *AI*. How many other segments can you find and draw on the geoboard that are parallel to $\overline{AI}$?



Name them: $\overline{FN}$, $\overline{BJ}$, $\overline{GO}$, $\overline{LT}$, $\overline{KS}$, $\overline{QY}$, $\overline{PX}$

2. A How many segments can you find that are **both** parallel and congruent to $\overline{AH}$?

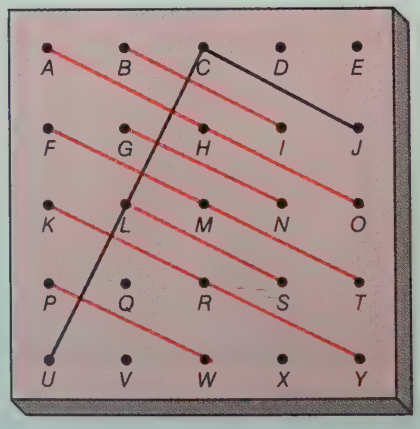


Name them: $\overline{BI}$, $\overline{CJ}$, $\overline{FM}$, $\overline{GN}$, $\overline{KR}$, $\overline{LS}$, $\overline{MT}$, $\overline{PW}$, $\overline{QX}$, $\overline{RY}$,

B Can you find and name some segments that are parallel to $\overline{AH}$, but **not** congruent to $\overline{AH}$?

Name them: $\overline{FT}$, $\overline{KY}$

3. Segment *CJ* is perpendicular to segment *CU*. How many other segments perpendicular to $\overline{CU}$ can you find and draw on this geoboard? (Assume that perpendicular segments touch each other.)

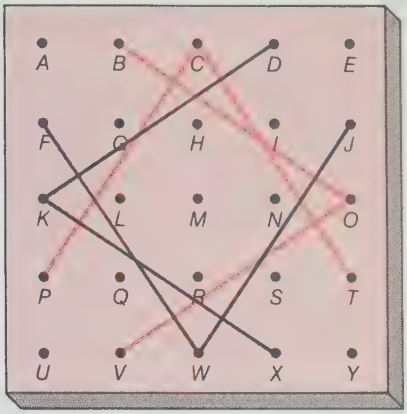


Name them: $\overline{CJ}$, $\overline{AH}$, $\overline{AO}$, $\overline{FM}$, $\overline{FT}$, $\overline{LS}$, $\overline{KR}$, $\overline{KY}$, $\overline{PW}$, $\overline{BI}$, $\overline{GN}$

Actually using geoboards or dot paper will be helpful for the children in completing these exercises.

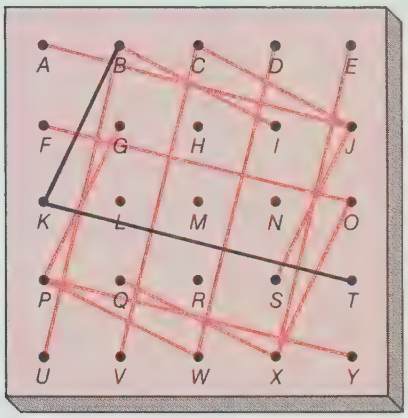
Congruent Angles

1. Angle *FWJ* is congruent to angle *DKX*. How many other angles can you find and draw on the geoboard that are congruent to ∠*DKX*?



Name them: ∠*FWJ*, ∠BOV, ∠PCT

2. How many angles can you draw on the geoboard that are congruent to ∠*BKT*?

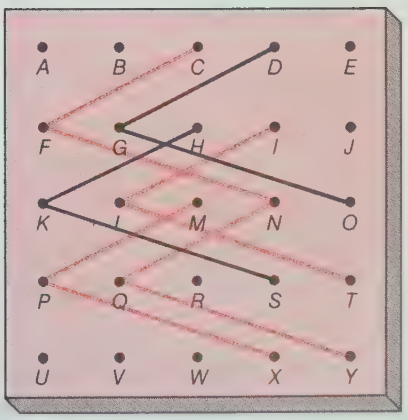


Name them: ∠GPY, ∠PWD, ∠QXE

∠XOF, ∠SJA, ∠JCV,

∠IBU

3. A ∠*DGO* is congruent to ∠*HKS*. Also, the sides of the two angles are parallel. How many other such angles (parallel sides) can you draw on the geoboard?



Name them: ∠*DGO*, ∠CFN, ∠ILT

∠MPX, ∠NQY

B Can you find 5 other angles that are congruent (but not parallel sides) to ∠*HKS*? Sample answers

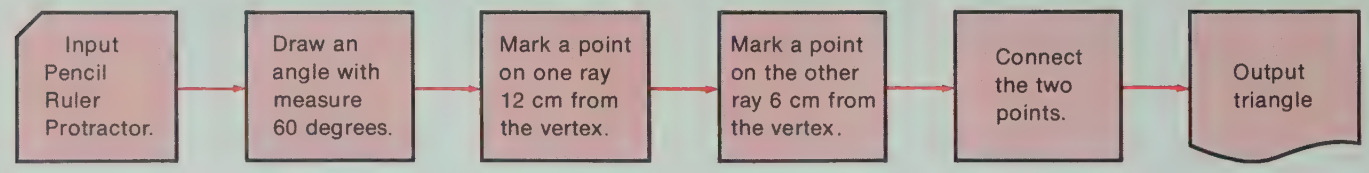
Name them: ∠KVH, ∠FQC, ∠LWI,

∠GRD, ∠MXL, ∠HSE

Using geoboards or dot paper will prove helpful in completing these exercises.

Angle Measure

1. Can you use the flow chart to draw a triangle in the space below?



2. Measure the other two angles. 90° 30°
If you did your work carefully, their measures should be about 90 degrees and 30 degrees.

3. Find the sum of the degree measures of the three angles. 180°
Your result should be very close to 180 degrees.

4. Draw any large triangle on another sheet of paper. Record the measure of the angles. → _____ _____ _____

Find the sum →

The sum should be very close to 180 degrees. _____

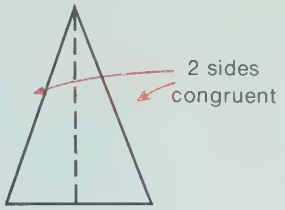
The general concept of this lesson is that the sum of the angles of any triangle is 180 degrees.

Symmetric Figures

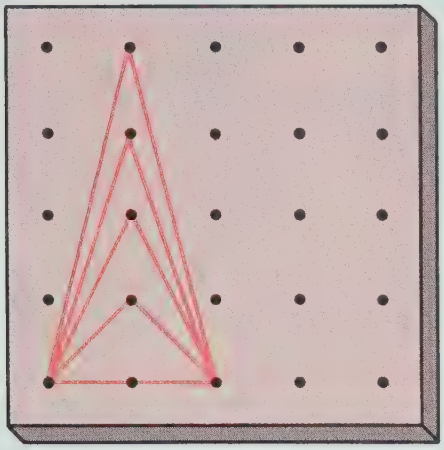
An isosceles triangle is a symmetrical figure.

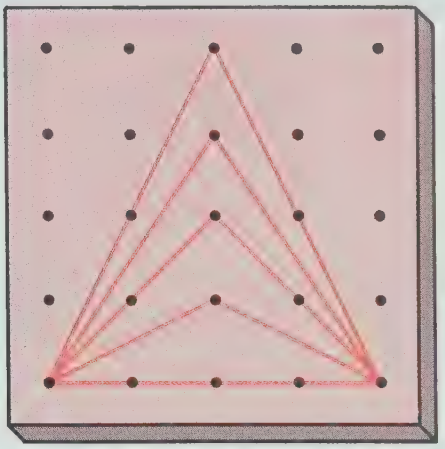
How many different shaped isosceles triangles can you find and draw on the geoboards below?

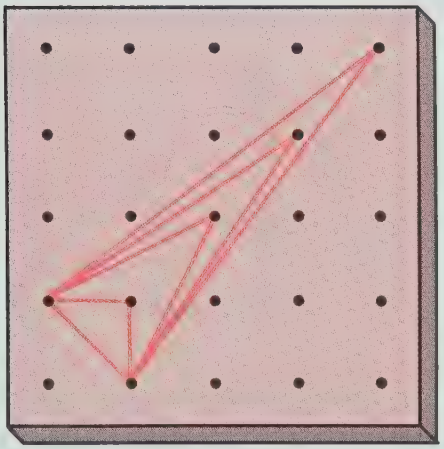
Don't quit too soon; there are 22.

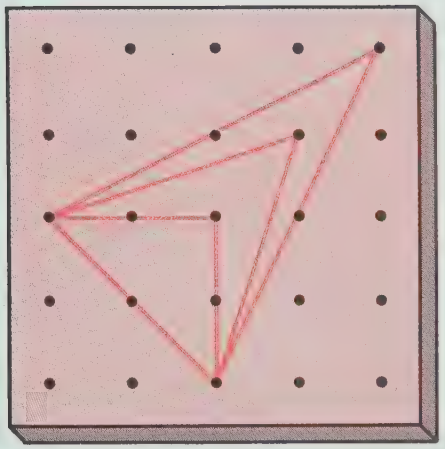


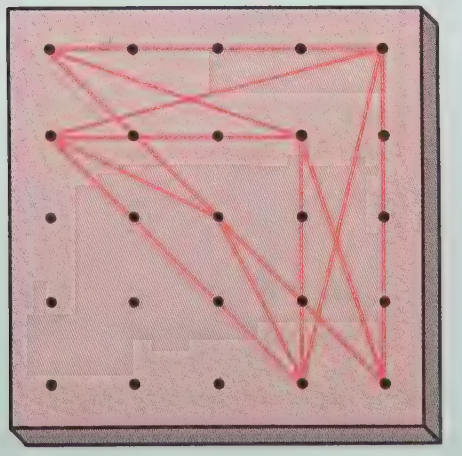
Isosceles Triangle

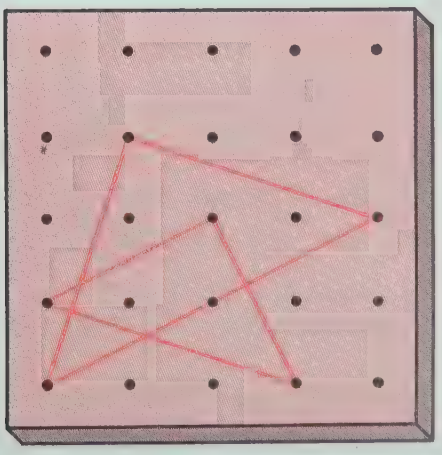








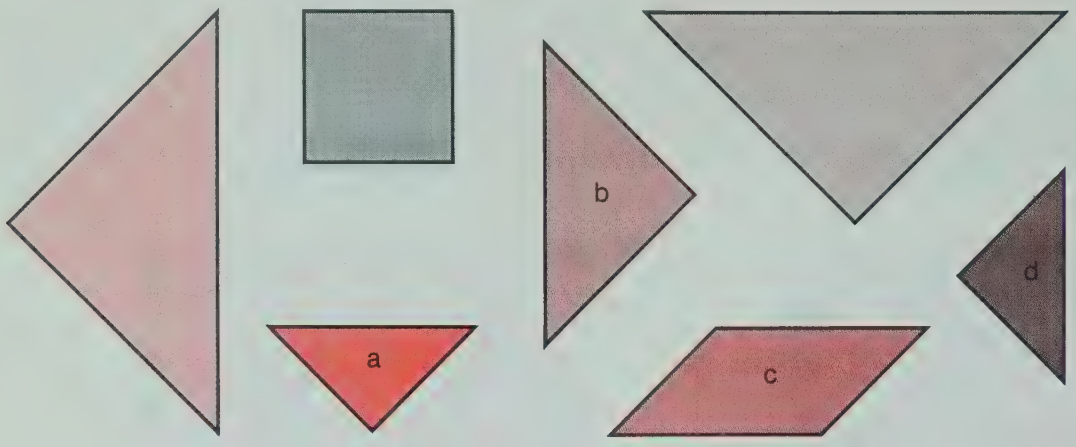




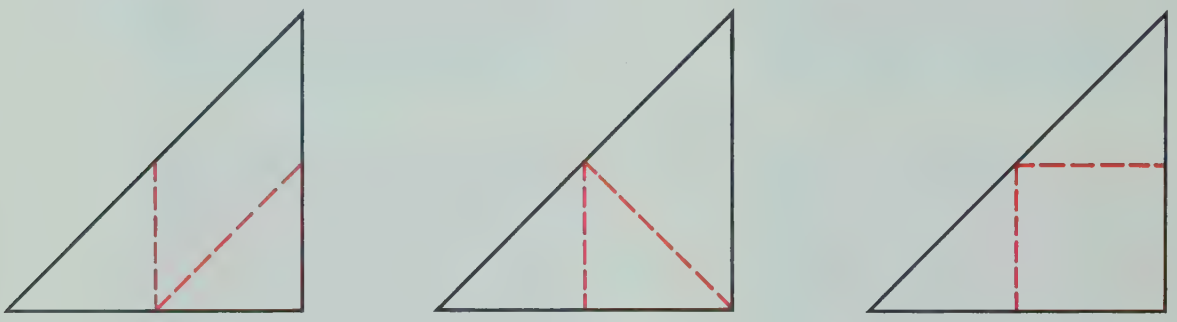
Using geoboards or dot paper will be helpful in completing these exercises.

Forming Congruent Figures

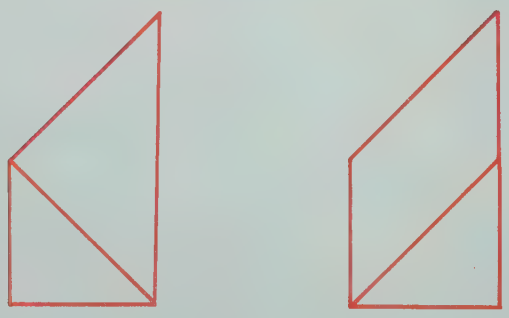
These are the 7 pieces of the tangram puzzle.



1. How many ways can you use the 5 small pieces to form a figure that is congruent to one of the large triangles? Record your findings by drawing the figures in the large triangles below.



2. Can you use pieces a, b, c, and d to form two congruent figures? Show how you did it in the space below. Sample answers

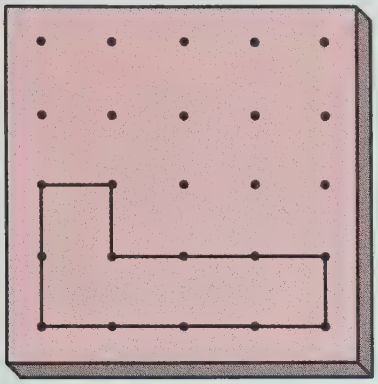


3. Using the 5 small pieces, form a pair of congruent isosceles trapezoids. Record how you did it in the two isosceles trapezoids below.

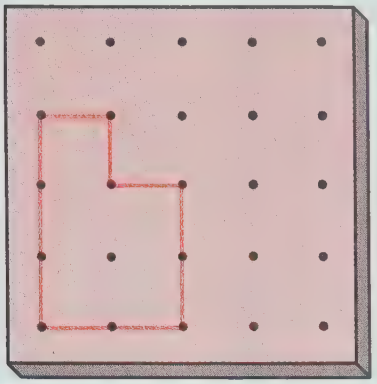
It may be helpful to show the children how to record their findings by working through one part of exercise 1 with them.

Perimeter and Area

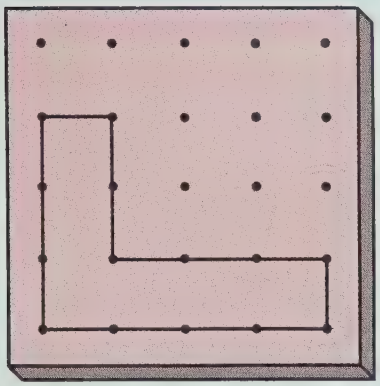
1. The perimeter of this figure is 12 units. Its area is 5 square units.



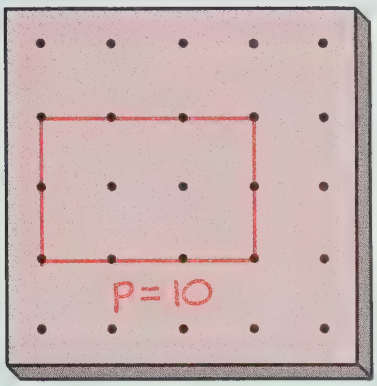
Can you show a figure on this geoboard that has perimeter 10 and area 5?

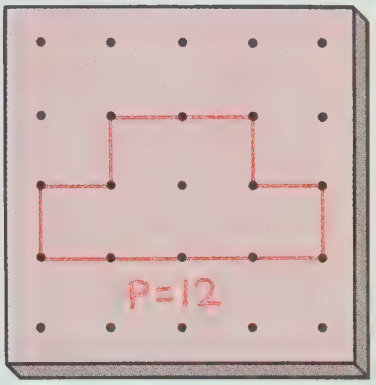


2. The area of this figure is 6 square units and its perimeter is 14.

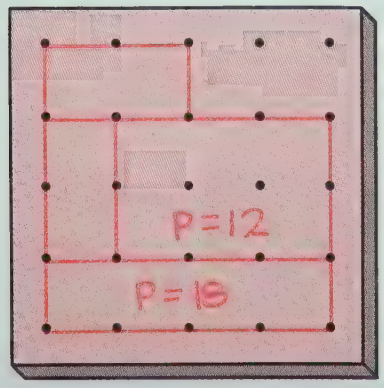


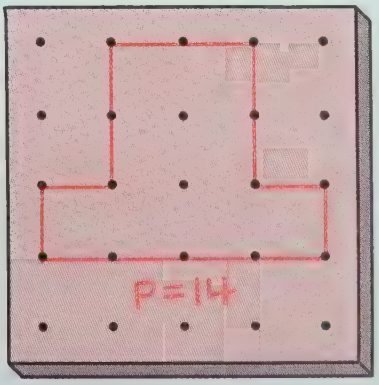
Can you show two other figures with area 6 but different perimeters?

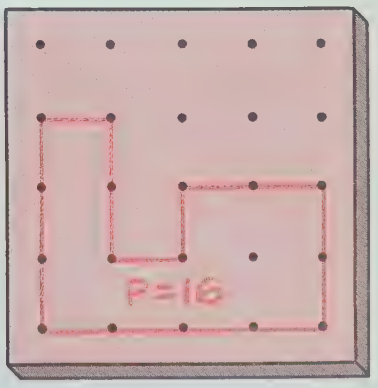




3. How many different perimeters can you show for figures that all have area 8? Four different ones are possible.



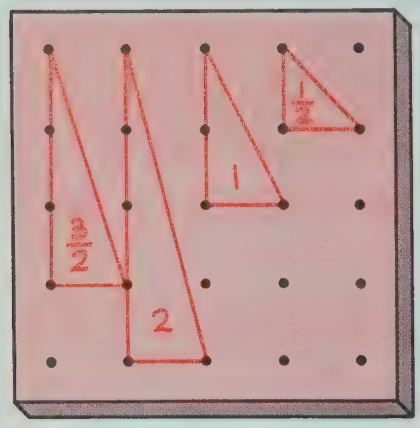


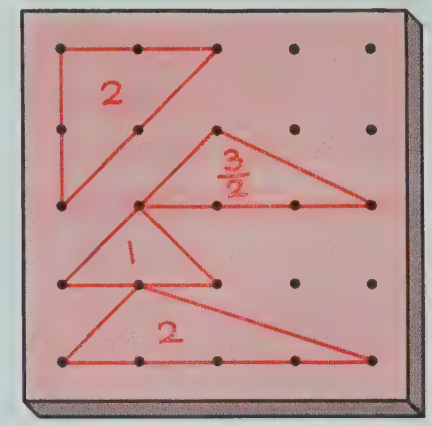


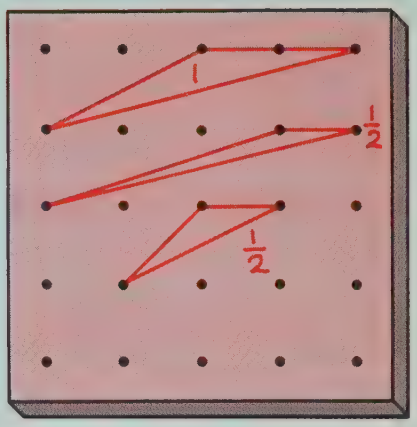
4. Try Exercise 3 for an area of 9 on another sheet of paper with 5 × 5 sets of dots.

Area of Triangles on the Geoboard

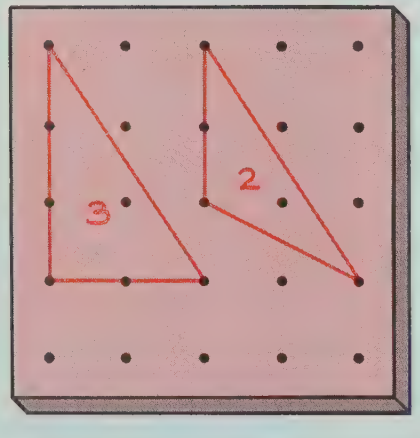
1. Using the dots as corners, how many different shaped triangles can you show that have no dots inside? Give the area of each one. Sample answers

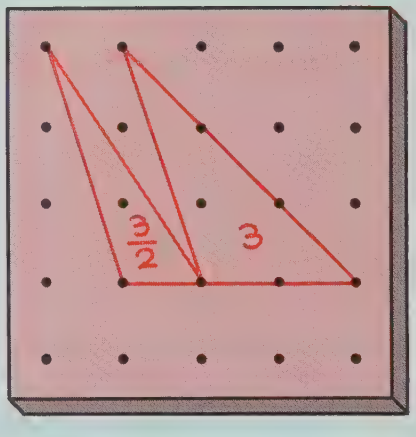


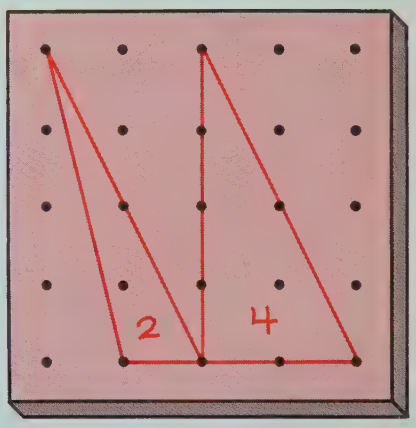




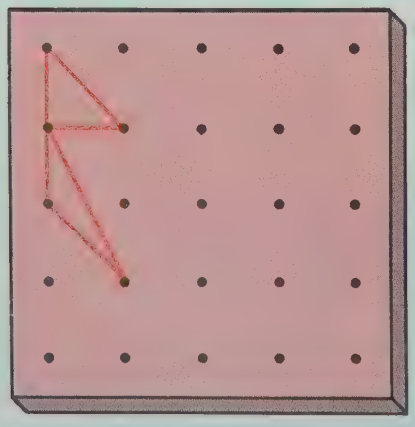
2. How many different shaped triangles can you show that have exactly one dot inside? Give the area of each one. Sample answers

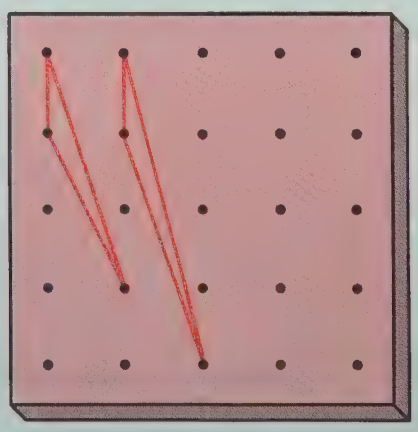


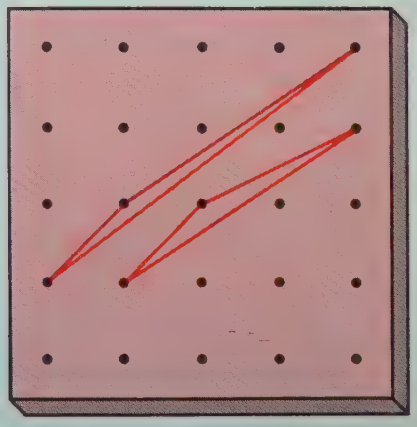




3. All the triangles that touch only 3 dots (on the corners only) have area $\frac{1}{2}$. How many of them can you find? (There are 6).







5 Estimation — Estimating

1. The numbers in set A are between 250 and 350. The numbers in set B are between 400 and 500.

A List five numbers in Set A. answers will vary

B List five numbers in Set B. answers will vary

C Give these numbers: largest in A 349; smallest in B 401;

largest in B 499; smallest in A 251.

D Make up some addition problems using one number from A and one from B in each problem. Estimate the sums. answers will vary

From A →
From B → +
Estimated Sum →

E What is the smallest sum you could get by adding one number from A and one from B? 652

F What is the largest such number? 848

G Use a colored pencil to show a set C on the number line above that contains all possible sums like those in Part D.

2. The numbers in Set D are between 3000 and 4000. The numbers in Set E are between 4500 and 5500.

A Make up some problems as you did in 1D above. answers will vary

From D →
From E → +
Estimated Sum →

B Show a Set F on the number line that contains all possible sums like those from part A.

Help children understand that the set containing all possible sums includes all the numbers from the smallest to the largest possible sum.

Special Products

1. Give missing factors in the equations. Make all equations different. answers may vary

A

3 × 500 = 1500
5 × 300 = 1500

B

4 × 700 = 2800
7 × 400 = 2800

C

6 × 900 = 5400
9 × 600 = 5400

D

6 × 400 = 2400
6 × 400 = 2400
2 × 1200 = 2400
12 × 200 = 2400

E

3 × 400 = 1200
4 × 300 = 1200
6 × 200 = 1200
2 × 600 = 1200

F

10 × 300 = 3000
3 × 1000 = 3000
6 × 500 = 3000
5 × 600 = 3000

G

2 × 9000 = 18,000
9 × 2000 = 18,000
3 × 6000 = 18,000
6 × 3000 = 18,000
10 × 1800 = 18,000
18 × 1000 = 18,000

H

2 × 18,000 = 36,000
18 × 2000 = 36,000
4 × 9000 = 36,000
9 × 4000 = 36,000
3 × 12,000 = 36,000
12 × 3,000 = 36,000

I

10 × 4000 = 40,000
40 × 1000 = 40,000
2 × 20,000 = 40,000
20 × 2000 = 40,000
50 × 8000 = 40,000
8 × 5000 = 40,000

2. Give missing numbers in the tables.

A

×	7000	70	7	700
4000	28,000,000	280,000	28,000	2,800,000
4	28,000	280	28	2800
400	2,800,000	28,000	2800	280,000
40	280,000	2800	280	28,000

B

×	5000	5	500	50
70	350,000	350	35,000	3500
7000	35,000,000	35,000	3,500,000	350,000
7	35,000	35	3500	350
700	3,500,000	3500	350,000	35,000

It may be helpful to work through part of Table A with the children to help them see how they can figure out some of the missing numbers from the numbers shown in the table.

Special Quotients

1. Give missing numbers in the equations. Make all equations different. answers will vary

A
1400 ÷ 20 = 70
1400 ÷ 2 = 700
1800 ÷ 20 = 90
1800 ÷ 200 = 9
1800 ÷ 30 = 60
1800 ÷ 300 = 6

B
3500 ÷ 50 = 70
3500 ÷ 500 = 7
2400 ÷ 60 = 40
2400 ÷ 600 = 4
2400 ÷ 800 = 3
2400 ÷ 300 = 8

C
7200 ÷ 900 = 8
7200 ÷ 90 = 80
3600 ÷ 40 = 90
3600 ÷ 400 = 9
3600 ÷ 2 = 1800
3600 ÷ 20 = 180

G
12,000 ÷ 300 = 40
12,000 ÷ 400 = 30
12,000 ÷ 4000 = 3
12,000 ÷ 3000 = 4
12,000 ÷ 200 = 60
12,000 ÷ 600 = 20

H
30,000 ÷ 600 = 50
30,000 ÷ 500 = 60
30,000 ÷ 3000 = 10
30,000 ÷ 1000 = 30
30,000 ÷ 2000 = 15
30,000 ÷ 1500 = 20

I
48,000 ÷ 600 = 80
48,000 ÷ 800 = 60
48,000 ÷ 1200 = 40
48,000 ÷ 4000 = 12
48,000 ÷ 2400 = 20
48,000 ÷ 2000 = 24

2. Give missing numbers in the tables.

A

×	40	100	6	30	90	600
20	800	2000	120	600	1800	12,000
50	2000	5000	300	1500	4500	30,000
300	12,000	30,000	1800	9000	27,000	180,000
4	160	400	24	120	360	2400

B

×	90	50	70	300	8	4000
8	720	400	560	2400	64	32,000
200	18,000	10,000	14,000	60,000	1600	800,000
20	1800	1000	1400	6000	160	80,000
60	5400	3000	4200	18,000	480	240,000

It may be helpful to work through part of Table A with the children to help them see how they can figure out some of the missing numbers from the numbers that are given.

Making Estimation Problems

An answer to each estimation problem is given first. You are to put numbers in the problem that make sense and make the answer a "good" estimate. Answers will vary

1. Answer: About 35,000 nuts.

There are 3500 nuts in each basket. How many nuts are there in all 10 baskets?

2. Answer: About $120,000.

Jim Jones earns 1000 dollars a month. At this rate, how much will he earn in 10 years?

3. Answer: 100,000 minutes.

Terri practiced the piano 100 minutes a day for 3 years. How many minutes was this in all?

4. Answer: About a million calories.

Many people eat about 3000 calories a day. How many calories is this in a year?

5. Answer: About 100,000 pages.
Judy figured she had read about 500 books in her life. If the books averaged 200 pages each, how many pages had she read?

6. Answer: About 5 million minutes.
There are 525,600 minutes in a year.

If Fritz is 10 years old, how many minutes old is he?

7. Answer: About $240,000.
The average cost of the cars in the lot is 2400 dollars. How much would all 100 cars be worth?

8. Answer: About 300,000 people.

The stadium holds 30,000 people. It was full for each of the 10 games. What was the total attendance?

9. Answer: About 20,000 kilometers

Juan walks about 5 kilometers each day. He is 10 years old. How far has he walked in his life?

10. Make a problem of your own for the answer below.
Answer: About 150,000

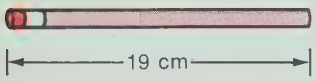
Encourage the children to be as creative as possible when substituting numbers in these problems. Attempt to get them to use realistic numbers rather than numbers containing a large number of zeros.

Estimating Metric Measures

Use the given fact to help you estimate each of the other measures using the given unit. Then, if you can, check to see how close you came.

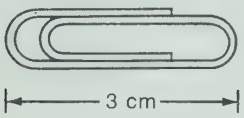
Answers will vary

1. A new pencil is about 19 centimeters long.



	Estimate	Actual
A How wide is your desk?	_____	_____
B How tall are you?	_____	_____
C How long is your pencil now?	_____	_____
D How long is your room?	_____	_____
E How tall is the door?	_____	_____

2. An ordinary paper clip is about 3 centimeters long.



A How thick is your math book?	_____	_____
B How wide is your math book?	_____	_____
C How far is it around your wrist?	_____	_____
D How long is your shoe?	_____	_____
E How wide is your hand?	_____	_____

3. It's about 950 kilometers from Chicago to Washington.

A How far is it from Chicago to Boston?	_____	_____
B How far is it from Chicago to New York?	_____	_____
C How far is it from New York to Washington?	_____	_____
D How far is it from New York to Boston?	_____	_____

4. A nickel weighs about 5 grams.

5 gm

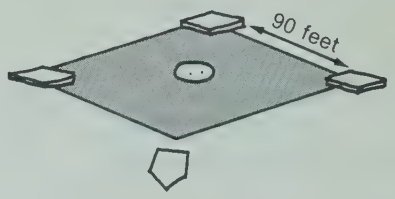
A How much does a penny weigh? (Be careful!)	_____	_____
B How much does a chalkboard eraser weigh?	_____	_____
C How much does a small paper clip weigh?	_____	_____
D How much does your ruler weigh?	_____	_____

Metric Measures in Sports

Many standard measures in sports are given in the English system of measure. Use the facts given to estimate the measures in metric units.

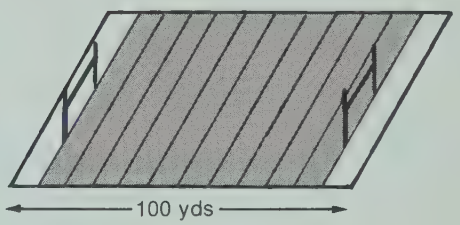
1. The distance between the bases in baseball is 90 feet. One meter is a little more than 3 feet.

Estimate the base distance in meters. 30



2. A football field is 100 yards long. 9 meters is almost 10 yards. Estimate the length of

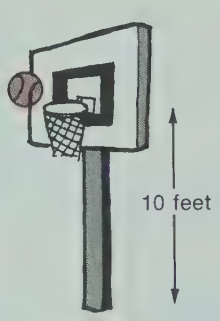
a football field in meters. 90



3. The height of the basket in basketball is 10 feet. There are about $30\frac{1}{2}$ centimeters in 1 foot. Estimate the basket height in

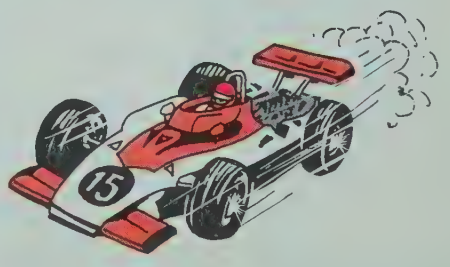
A meters. 3

B centimeters. 300



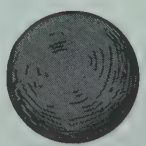
4. One of the most famous of all auto races is the Indianapolis 500. The cars race for 500 miles. There are about 8 kilometers for each 5 miles.

Estimate the length of the race in kilometers. 800



5. The standard weight for the Olympic shot put is 16 pounds. One kilogram is about $2\frac{1}{5}$ pounds. Estimate

the weight of the shot put in kilograms. 7



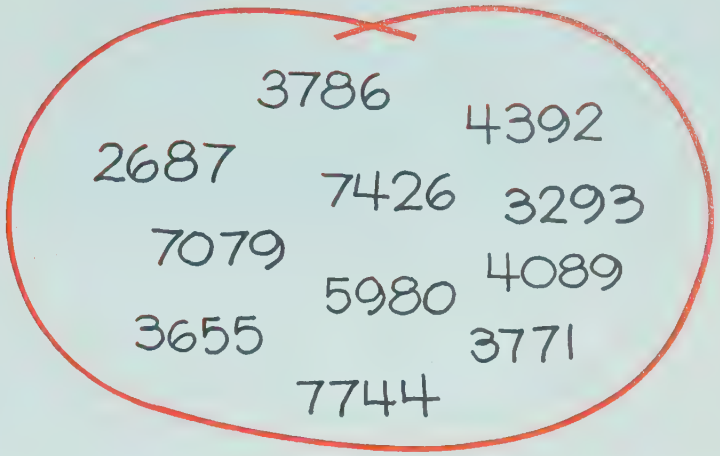
16 pounds

The children **should not** be encouraged to make exact conversions from English to Metric. Rather, they should work solely with the given information to estimate the metric measures.

How many addition and subtraction problems can you write using numbers from this set.

Sample:

$$\begin{array}{r} 3293 \\ +\ 2687 \\ \hline 5980 \end{array}$$



Sample answers

$\begin{array}{r} 2687 \\ +\ 3293 \\ \hline 5980 \end{array}$	$\begin{array}{r} 5980 \\ -\ 3293 \\ \hline 2687 \end{array}$	$\begin{array}{r} 5980 \\ -\ 2687 \\ \hline 3293 \end{array}$	$\begin{array}{r} 3293 \\ +\ 3786 \\ \hline 7079 \end{array}$	$\begin{array}{r} 7079 \\ -\ 3786 \\ \hline 3293 \end{array}$
$\begin{array}{r} 7079 \\ -\ 3293 \\ \hline 3786 \end{array}$	$\begin{array}{r} 2687 \\ +\ 4392 \\ \hline 7079 \end{array}$	$\begin{array}{r} 7079 \\ -\ 4392 \\ \hline 2687 \end{array}$	$\begin{array}{r} 7079 \\ -\ 2687 \\ \hline 4392 \end{array}$	$\begin{array}{r} 3655 \\ +\ 3771 \\ \hline 7426 \end{array}$
$\begin{array}{r} 7426 \\ -\ 3655 \\ \hline 3771 \end{array}$	$\begin{array}{r} 7426 \\ -\ 3771 \\ \hline 3655 \end{array}$	$\begin{array}{r} 4089 \\ +\ 3655 \\ \hline 7744 \end{array}$	$\begin{array}{r} 7744 \\ -\ 3655 \\ \hline 4089 \end{array}$	$\begin{array}{r} 7744 \\ -\ 4089 \\ \hline 3655 \end{array}$

Adding and Subtracting with Metric Units

Adding and subtracting is much the same with meters and centimeters as it is with dollars and cents.

Complete the table.

Cents	Dollars	Cents	We write	We read
275	2	75	$2.75	two dollars and seventy five cents
362	3	62	$3.62	three dollars and sixty two cents
429	4	29	$4.29	four dollars and twenty nine cents

There are 100 cents in 1 dollar.
There 100 centimeters in 1 meter.

Now complete this table.

Centimeters	Meters	Centimeters	We write	We read
275	2	75	2.75 meters	two and seventy five hundredths meters
362	3	62	3.62 meters	three and sixty two hundredths meters
429	4	29	4.29 meters	four and twenty nine hundredths meters

Note: For 2.75 meters, you can also read, "Two meters and 75 centimeters."

1. Find the totals.

A			B			C		
$3.4 6	3.4 6 meters		$5.2 7	5.2 7 meters		$3.8 5	3.8 5 meters	
2.8 2	2.8 2 meters		6.9 5	6.9 5 meters		2.9 8	2.9 8 meters	
$6.28	6.28 meters		$12.22	12.22 meters		$6.83	6.83 meters	

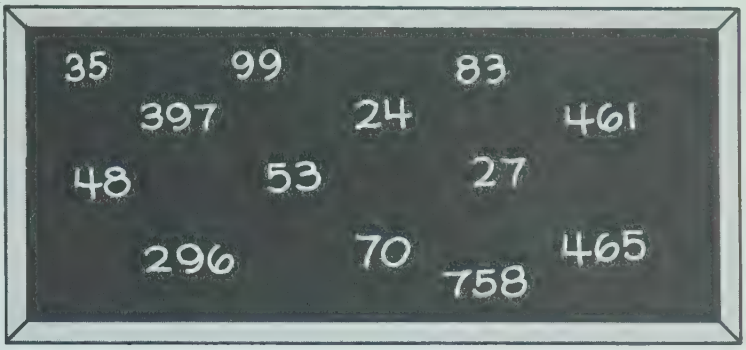
2. Find the differences in the amounts.

A			B			C		
$4.2 5	4.2 5 meters		$6.2 8	6.2 8 meters		$5.3 0	5.3 0 meters	
1.7 5	1.7 5 meters		1.3 5	1.3 5 meters		2.8 9	2.8 9 meters	
$2.50	2.50 meters		$4.93	4.93 meters		$2.41	2.41 meters	

The goal here is to help children see that working with meters and centimeters using decimals is exactly the same as working with dollars and cents using decimals.

Multiplying

Can you find the two factors for the multiplication problems below. They are all on the chalkboard. Look for clues such as: The product of two odd numbers is odd; a factor ending in 5 will have a product ending in 5 or 0; the product of two 2-digit numbers can't have more than 4 digits—etc.



35	70	296	83	465
× 27	× 99	× 53	× 27	× 48
245	630	888	581	3720
700	6300	14,800	1660	18,600
945	6930	15,688	2241	22,320

397	461	99	758	83	70
× 35	× 24	× 48	× 24	× 53	× 35
1985	1844	792	3032	249	350
11,910	9220	3960	15,160	4150	2100
13,895	11,064	4752	18,192	4399	2450

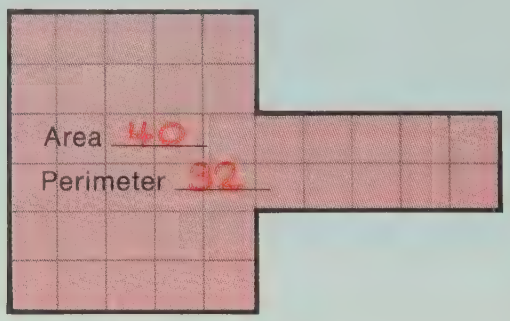
461	296	461	758	465	397
× 465	× 758	× 461	× 465	× 397	× 296
2305	2368	461	3790	3255	2382
27,660	14,800	27,660	45,480	41,850	35,730
184,400	207,200	184,400	303,200	139,500	79,400
214,365	224,368	212,521	352,470	184,605	117,512

Point out to the children that in addition to some of the clues given in the directions they should use estimation to help them decide on the two factors.

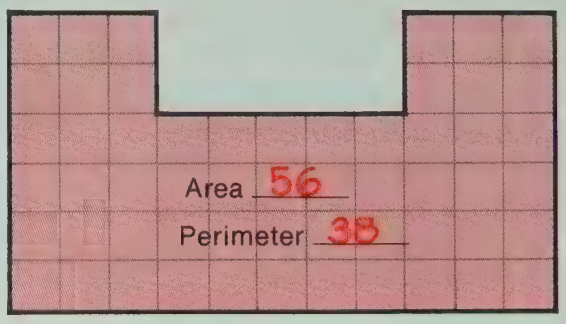
Computing – Area and Perimeter

1. Give the area and perimeter of each figure.

A



B



2. Draw figures with the given area and perimeter. Shapes may vary

A

Area 63

Perimeter 32

B

Area 48

Perimeter 38

C

Area 154

Perimeter 50

D

Area 118

Perimeter 50

1-Digit Divisors

First write your estimate of the quotient in the colored box. Then find the quotient. Next find the difference between the two numbers. Rate yourself as directed below.

1. $4\overline{)1496}$ = 374

Estimate

Difference

Score

2. $6\overline{)1716}$ = 286

Estimate

Difference

Score

3. $9\overline{)4149}$ = 461

Estimate

Difference

Score

4. $7\overline{)3689}$ = 527

Estimate

Difference

Score

5. $8\overline{)6896}$ = 862

Estimate

Difference

Score

6. $5\overline{)3770}$ = 754

Estimate

Difference

Score

7. $7\overline{)2765}$ = 395

Estimate

Difference

Score

8. $9\overline{)5472}$ = 608

Estimate

Difference

Score

9. $8\overline{)7296}$ = 912

Estimate

Difference

Score

Score each difference as follows:

- 10 or less—5 points
- Between 10 and 50—3 points
- 50 to 100—1 point
- Over 100—0 points

Add your scores and rate yourself.

- 35–45 Fantastic
- 25–34 Excellent
- 15–24 Good
- 0–14 Keep trying

To be sure children rate themselves accurately, see that they estimate each quotient before they actually find the quotient.

Finding Averages

The little league Giants played ten games last summer. Here are the scores for the games. The scores are not listed in the order the games were played.

Giants 12	W	Cubs 6
Giants 1	L	Dodgers 7
Giants 3	W	Yankees 2
Giants 7	W	White Sox 6
Giants 14	W	Cardinals 10
Giants 6	W	Tigers 2
Giants 4	L	Reds 7
Giants 8	L	Red Sox 9
Giants 10	W	Braves 9
Giants 5	W	Athletics 2

1. The Giants lost their first two games to the Dodgers and the Red Sox. Then they won their next game. For all three games they averaged 5 runs per game. Who did they play?

Tigers (scored 6 runs)

2. The Giants scored enough runs in the fourth game to keep their average at 5 runs per game. What team did they play? Athletics

(scored 5 runs in the 4th game)

3. In the fifth game they increased their average to 6 runs per game. What team did they play?

Braves

(scored 10 runs in the 5th game)

4. What was their average number of runs for the last five games of the season?

$$5\overline{)40}$$

8

8 runs per game

5. Against what 3 teams did they average 12 runs per game?

Cubs, Cardinals, Braves

6. Against what 4 teams did they average only about 3 runs per game?

Dodgers, Yankees, Reds, Athletics

7. How many runs per game did the Giants average for the season? (10 games) 7 runs per game

8. How many runs per game did their opponents average for the season?

6 runs per game

For exercise 6 children may have to use some logic to find the answer. Since the average does not turn out exact, they will be looking for the four games in which the Giants had the lowest scores.

"Short Division"

1. Can you find the missing divisors?

A $7\overline{)3254}$ = 464 R6

B $6\overline{)2714}$ = 452 R2

C $5\overline{)3726}$ = 745 R1

D $9\overline{)6032}$ = 670 R2

E $8\overline{)4287}$ = 535 R7

F $4\overline{)2652}$ = 663 R0

G $3\overline{)2717}$ = 905 R2

H $7\overline{)3846}$ = 549 R3

I $6\overline{)5127}$ = 854 R3

2. Can you find the missing dividend?

A $7\overline{)1724}$ = 246 R2

B $5\overline{)688}$ = 137 R3

C $9\overline{)3859}$ = 428 R7

D $7\overline{)1793}$ = 256 R1

E $4\overline{)2430}$ = 607 R2

F $6\overline{)2892}$ = 482 R0

G $7\overline{)4583}$ = 654 R5

H $8\overline{)2319}$ = 289 R7

I $5\overline{)2033}$ = 406 R3

3. Give missing numbers so the statement is true.

A The average of this set of numbers is 23.

18, 27, 20, 29, 12, 32

B The average of this set of numbers is 52.

39, 64, 56, 48, 71, 36, 50

C The average of this set of numbers is 238.

272, _____, 157, _____

Answers will vary

D The average of this set of numbers is 417.

_____, _____, _____, _____, _____

Answers will vary

In exercise 1, encourage the children to use reasoning rather than long division to find the divisors. Give the children an opportunity to discover how to find the dividend in exercise 2.

Estimating 1-Digit Quotients

Your ability to find 1-digit quotients quickly and accurately is the most important skill in finding larger quotients. First estimate each 1-digit quotient in the page **quickly,** then find the quotient.

1. $28\overline{)168}$ — 6 — Estimate

2. $53\overline{)477}$ — 9 — Estimate

3. $41\overline{)164}$ — 4 — Estimate

4. $59\overline{)413}$ — 7 — Estimate

5. $37\overline{)222}$ — 6 — Estimate

6. $62\overline{)434}$ — 7 — Estimate

7. $79\overline{)711}$ — 9 — Estimate

8. $24\overline{)192}$ — 8 — Estimate

9. $56\overline{)168}$ — 3 — Estimate

10. $49\overline{)294}$ — 6 — Estimate

11. $96\overline{)480}$ — 5 — Estimate

12. $35\overline{)280}$ — 8 — Estimate

13. $64\overline{)256}$ — 4 — Estimate

14. $77\overline{)385}$ — 5 — Estimate

15. $45\overline{)360}$ — 8 — Estimate

Rate yourself:
14 or 15 right—outstanding
12 or 13 right—excellent
10 or 11 right—good
8 or 9 right—fair
7 or less right—need more practice

Be sure the children find all of the estimates first before actually determining the one-digit quotient.

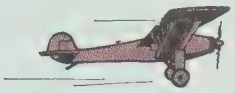
How Are You Traveling?

Suppose you travel very close to the average speeds given below in kilometers per hour (km/h).


Walking
4 km/h


Bicycling
15 km/h


Car
88 km/h


Small plane
220 km/h


Jet plane
900 km/h

Use estimation to decide how you would be traveling under the given conditions.

1. New York to Chicago

Distance: 1344 km
Time: 6 hours

Traveled how? Small plane

2. Miami to Key West

Distance: 251 km
Time: 17 hours

Traveled how? Bicycle

3. Kansas City to St. Louis

Distance: 403 km
Time: $4\frac{1}{2}$ hours

Traveled how? Car

4. Boston to Toronto

Distance: 720 km
Time: Almost an hour

Traveled how? Jet plane

5. Washington to Baltimore

Distance: 61 km
Time: 15 hours

Traveled how? Walking

6. Detroit to Chicago

Distance: 435 km
Time: 2 hours

Traveled how? Small plane

7. Chicago to Milwaukee

Distance: 145 km
Time: 10 hours

Traveled how? Bicycle

8. Minneapolis to St. Paul

Distance: 14 km
Time: $3\frac{1}{2}$ hours

Traveled how? Walking

9. San Francisco to Dallas

Distance: 2820 km
Time: 3 hours 15 min.

Traveled how? Jet plane

10. Indianapolis to Columbus, Ohio

Distance: 276 km
Time: 3 hours 15 min.

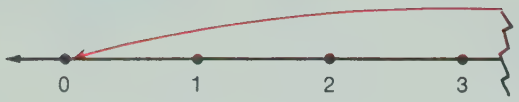
Traveled how? Car

Encourage the children to use as much logic as possible in determining these answers. Pencil and paper computation are not needed for these exercises.

Number Line Jumps

First make an estimate. Then check your estimate by computing. Estimates will vary

1.



How many jumps?

		Estimate	Actual
A	If each jump is 37?	_____	42
B	If each jump is 42?	_____	37
C	If each jump is 74?	_____	21
D	If each jump is 21?	_____	74

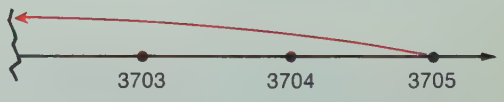
2.

How many jumps?

		Estimate	Actual
A	If each jump is 91?	_____	33
B	If each jump is 39?	_____	77
C	If each jump is 77?	_____	39
D	If each jump is 21?	_____	143

3.

How long is each jump?



		Estimate	Actual
A	If 65 jumps are taken?	_____	57
B	If 95 jumps are taken?	_____	39
C	If 57 jumps are taken?	_____	65
D	If 39 jumps are taken?	_____	95

Making Division Problems

Make your own division problems according to directions. Complete the dividing to check your problem.

1. 1-digit divisor
1-digit quotient
Remainder 0

$6 \overline{)\,54} = 9$ R0

2. 2-digit divisor
1-digit quotient
Remainder 10

$15 \overline{)\,70} = 4$ R10

3. 1-digit divisor
2-digit quotient
Remainder 5

$6 \overline{)\,71} = 11$ R5

Sample answer

4. 2-digit divisor
1-digit quotient
Remainder 12

$14 \overline{)\,82} = 5$ R12

Sample answer

5. 1-digit divisor
1-digit quotient
Remainder 7

$9 \overline{)\,79} = 8$ R7

Sample answer

6. 2-digit divisor
2-digit quotient
Remainder 0

$12 \overline{)\,168} = 14$ R0

Sample answer

7. 2-digit divisor
2-digit quotient
Remainder 23

$25 \overline{)\,498} = 19$ R23

8. Multiple of ten divisor
2-digit quotient
Remainder 6

$30 \overline{)\,876} = 29$ R6

Sample answer

It may be helpful to have children try sample divisors for these exercises in order to arrive at the correct divisor and quotient.

Making Special Division Problems

You have just two rules to follow when you make these problems.

Rule 1 The divisor and the quotient are the same.
Rule 2 The remainder is zero.

1. $6 \overline{)36}$ R0

2. $8 \overline{)64}$ R0

3. $10 \overline{)100}$ R0

4. $12 \overline{)144}$ R0

5. $15 \overline{)225}$ R0

6. $19 \overline{)361}$ R0

7. $25 \overline{)625}$ R0

8. $29 \overline{)841}$ R0

9. $38 \overline{)1444}$ R0

10. $53 \overline{)2809}$ R0

The answers to these exercises can be found by logical reasoning. Children are not expected to take the square root of the dividend to find the quotient and the divisor.

Finding the Missing Digits

Find the missing digits.

1.

```
    4 7
  ------
6 ) 2 8 5
    2 4
  ------
      4 5
      4 2
  ------
        3
```

2.

```
      8 9
    -----
4 ) 3 5 7
    3 2
    -----
      3 7
      3 6
    -----
        1
```

3.

```
      2 4 3
    -------
7 ) 1 7 0 7
    1 4
    -------
      3 0
      2 8
    -------
        2 7
        2 1
    -------
            6
```

4.

```
        3 5
    -------
4 2 ) 1 4 8 0
      1 2 6
    -------
        2 2 0
        2 1 0
    -------
          1 0
```

5.

```
        6 2
    -------
5 7 ) 3 5 3 4
      3 4 2
    -------
          1 1 4
          1 1 4
    -------
              0
```

6.

```
        7 3
    -------
8 4 ) 6 1 5 0
      5 8 8
    -------
        2 7 0
        2 5 2
    -------
          1 8
```

7.

```
      1 3 2
    -------
5 8 ) 7 6 7 6
      5 8
    -------
      1 8 7
      1 7 4
    -------
          1 3 6
          1 1 6
    -------
            2 0
```

8.

```
        3 8 2
    ---------
7 5 ) 2 8 6 6 0
      2 2 5
    -------
          6 1 6
          6 0 0
    -------
            1 6 0
            1 5 0
    -------
              1 0
```

9.

```
        5 4 8
    ---------
6 9 ) 3 7 8 1 2
      3 4 5
    -------
        3 3 1
        2 7 6
    -------
          5 5 2
          5 5 2
    -------
              0
```

Solving Problems

Suppose everyone in your class brought cookies for a bake sale. Give some numbers that make sense for the blanks below. Then write some problems about the sale. For each problem use one or more operations including the one suggested in the box.

Answers will vary

Total number of cookies to sell. _____

Number in your class. _____

Number of cookies in a box. _____

Price per box. _____

Number of cookies in a sack. _____

Price per sack. _____

Number in a dozen. _____

Price per dozen. _____

1. 

2. 

3. 

4. 

Help the children be realistic in terms of the numbers they use for this page. However, give them as much freedom as possible when they construct the four different problems.

Checking Up on Numbers

Study the example and complete the table. List all of the factors of the numbers shown in black except the number itself. Notice that when the sum of these factors is less than the given number it is called a **deficient** number. When the sum of these factors is equal to the number it is called **perfect**. And when the sum of these factors is greater than the number it is called an **abundant** number.

Number	Factors of the number (not including the number)	Sum of the factors of the number	Type of number
4	1, 2	3	deficient
6	1, 2, 3	6	perfect
8	1, 2, 4	7	deficient
9	1, 3	4	deficient
10	1, 2, 5	8	deficient
12	1, 2, 3, 4, 6	16	abundant
14	1, 2, 7	10	deficient
15	1, 3, 5	9	deficient
16	1, 2, 4, 8	15	deficient
18	1, 2, 3, 6, 9	21	abundant
20	1, 2, 4, 5, 10	22	abundant
21	1, 3, 7	11	deficient
22	1, 2, 11	14	deficient
24	1, 2, 3, 4, 6, 8, 12	36	abundant
26	1, 2, 13	16	deficient
28	1, 2, 4, 7, 14	28	perfect

It is not important that children remember the categories of numbers. Some children may be interested to know that very few perfect numbers have been discovered. For example, the twenty-third perfect number to be discovered contains nearly 6800 decimal digits.

1 Sorting Out the Primes

Follow these directions for the numbers in the red box.

A Mark out 1. It is not prime.

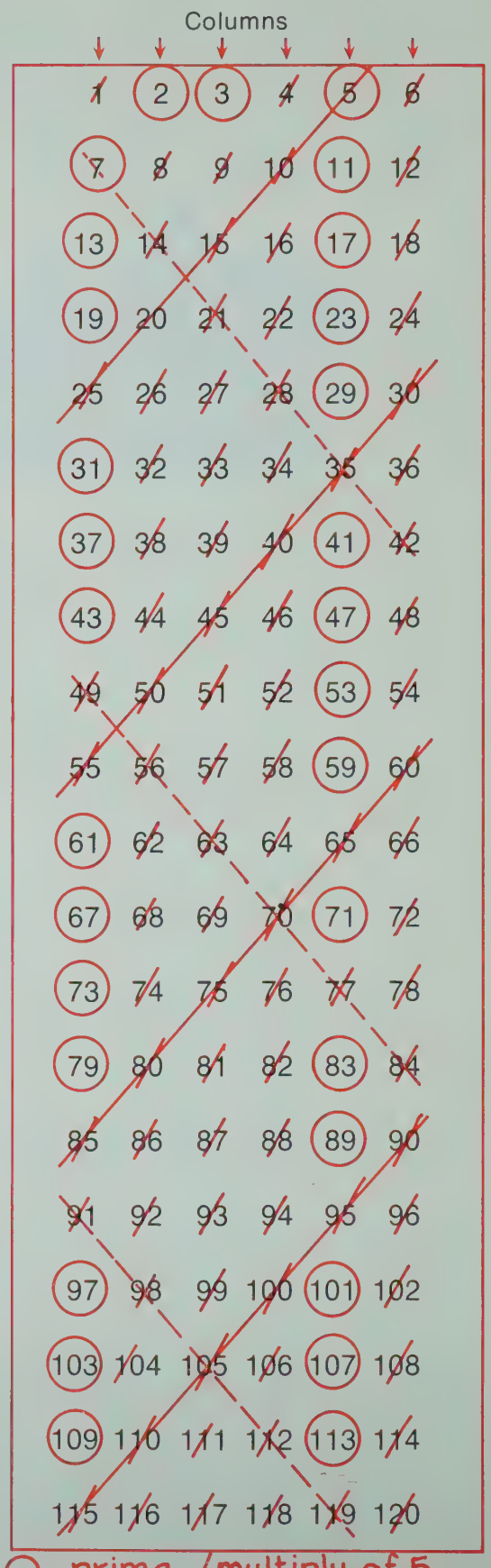
B 2 is prime. Each other multiple of 2 is not prime. Mark out each one.

C 3 is prime. Each other multiple of 3 is not prime. Mark out each one.

D 5 is prime. Each other multiple of 5 is not prime. Mark out each one.

E 7 is prime. Each other multiple of 7 is not prime. Mark out each one.

The numbers not marked out are the prime numbers less than 120. How many are there? 30



2 Searching for Patterns

A Do the multiples of 2 all lie in certain columns? Which columns? 2,4,6

B Do the multiples of 3 all lie in certain columns? Which columns? 3,6

C Do the multiples of 5 all lie according to some pattern? Draw a red line through these multiples to show the pattern.

D Do the multiples of 7 all lie according to some pattern? Draw a blue line through these multiples to show the pattern.

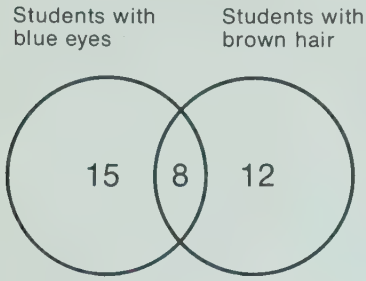
E Do all the prime numbers lie in certain columns? Which columns? 1, and 5 except for prime numbers 2 and 3

In recognizing patterns it might be helpful if children use the different colors for each of the marking out exercises. It will also be helpful if children circle each of the prime numbers.

Diagram Decisions

Each diagram below deals with sets that intersect. Yet each diagram is different in some way. Can you answer the questions? The numbers tell how many students are in each region of the diagram.

1.



How many students

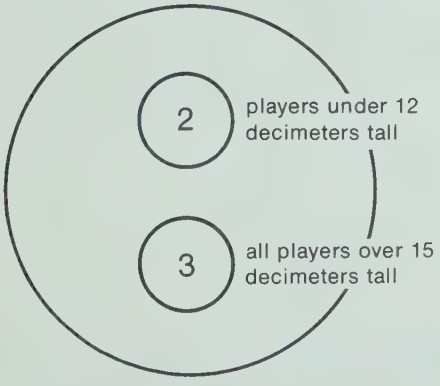
A have blue eyes? 23

B have brown hair? 20

C have blue eyes and brown hair? 8

D altogether? 35

2. All players on the basketball squad (17)



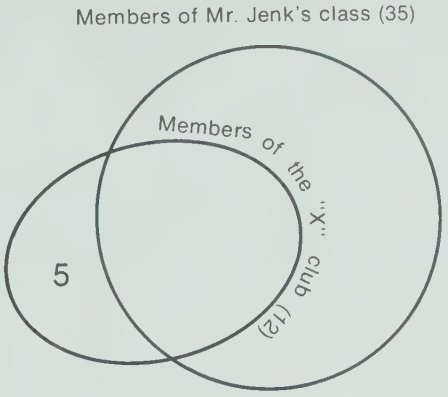
How many players are

A from 12 decimeters to 15 decimeters tall? 12

B 12 decimeters tall or over? 15

C 15 decimeters tall or under? 14

3.

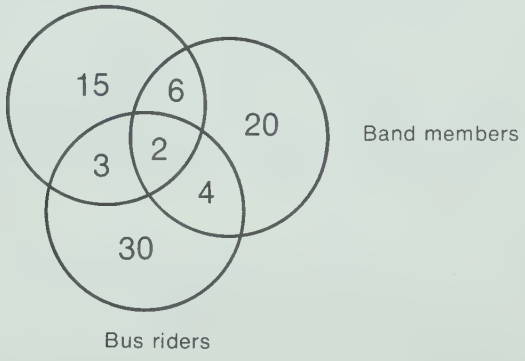


How many students in

A the "X" club are in Mr. Jenk's class? 7

B Mr. Jenk's class are not in the "X" club? 28

4. Mr. Jones' class



How many students

A in Mr. Jones' class? 26

B play in the band and ride the bus? 6

C in Mr. Jones' class ride the bus and play in the band? 2

D altogether? 80

It might be helpful to follow up this lesson with a discussion of each diagram having the children point out their reasoning in arriving at conclusions.

Let's Play a Factor Game

Play this "factor game" with a classmate. Use this **Number Chart,** the **scoring table,** and the **procedure** below.

2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17
18	19	20	21	22	23	24	25
26	27	28	29	30	31	32	33
34	35	36	37	38	39	40	41

Number Chart

Player A	Player B

Scoring Table

Procedure:

1. Player A picks a number from the chart, crosses it out and records it in the scoring table.
2. Player B crosses out each factor of A's number (not already crossed out) and records the sum of those factors in his column of the scoring table.
3. Player A picks another number, crosses it out in the chart, and records it in the table.
4. Since A's number has no factors in the table, it is now B's turn to choose a number, cross it out in the chart, and record it in his column.

EXAMPLE

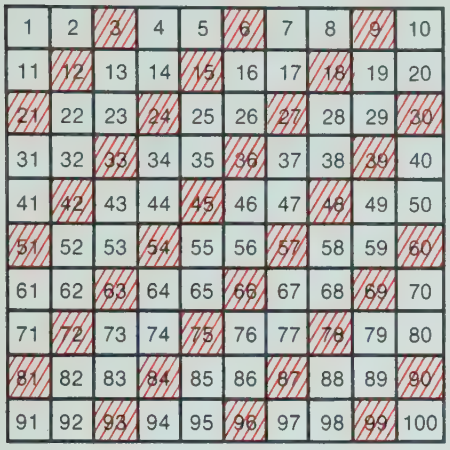
Player A	Player B
21	10 (3 + 7)
41	35
5	

5. The game proceeds as before with A crossing out all the factors of B's number (not already crossed out) and recording the sum of those numbers he crossed out in his column.
6. The player with the largest total score when all numbers are crossed out wins the game.

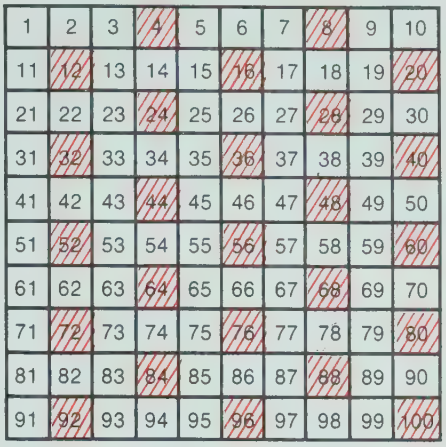
Children may need some guidance in beginning this game. They may discover that there are a number of strategies that can be taken to score the most points. For example, they should avoid choosing abundant numbers. When they choose a prime it becomes their opponents turn. However, choosing a large prime might be advantageous.

Multiple Patterns

Color each array according to the directions. Search for color patterns and then answer the questions.

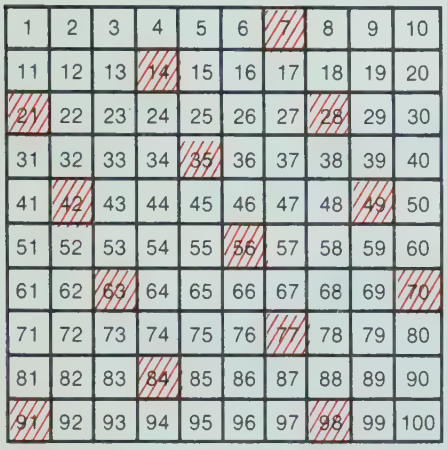


Color all multiples of 3

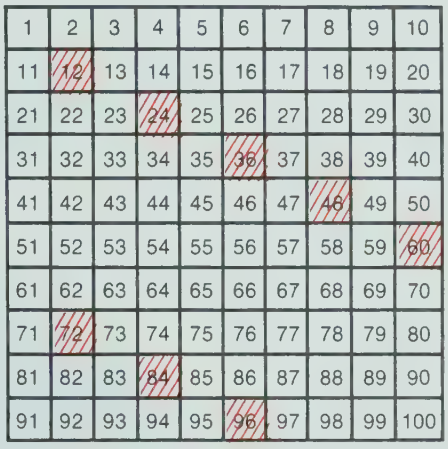


Color all multiples of 4

Color all multiples of 5



Color all multiples of 7



Color all multiples of 12

Color all multiples of 18

1. Which sets of multiples give column patterns? Multiples of 5

2. Which sets of multiples give diagonal patterns hitting all columns? Multiples of 3

3. Which sets of multiples give diagonal patterns hitting only certain columns? Multiples of 4, 7, 12 and 18

4. Use tracing paper to trace and fill in the coloring for multiples of 12. Place your tracing over the coloring for multiples of 18. How can you describe

   A the numbers that are colored twice? They are common multiples of 12 and 18.

   B the smallest of these numbers? It is the least common multiple of 12 and 18.

Try these activities and answer the questions.

1. Use a meter stick.
   - A Cut a piece of string 1 meter long. (Call it piece A.)
   - B Cut a piece of string $\frac{1}{10}$ meter long. (Call it piece B.)
   - C Can you cut a piece of string $\frac{1}{100}$ meter long? (Call this length of string piece C.)
   - D Can you cut a piece of string $\frac{1}{1000}$ meter long? (Call this length of string piece D.)

2. Use the string you cut in part 1. Write a fraction in each blank.

   A Piece B is $\frac{1}{10}$ piece A.

   B Piece C is $\frac{1}{10}$ piece B.

   C Piece D is $\frac{1}{10}$ piece C.

   D Piece C is $\frac{1}{100}$ piece A.

   E Piece D is $\frac{1}{1000}$ piece A.

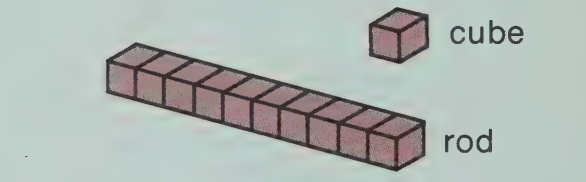
   F Piece D is $\frac{1}{100}$ piece B.

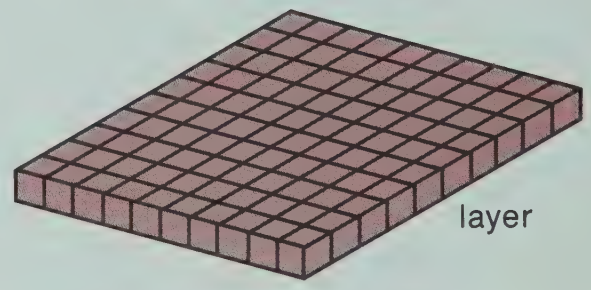
3. Use or think about 1 centimeter cubes.

   A It takes 10 of them to make a rod.

   B It takes 100 of them to make a layer.

   C It takes 1000 of them to make a block.





4. Think about the figures in part 3. Write a fraction in each blank.

   A A cube is $\frac{1}{10}$ of a rod.

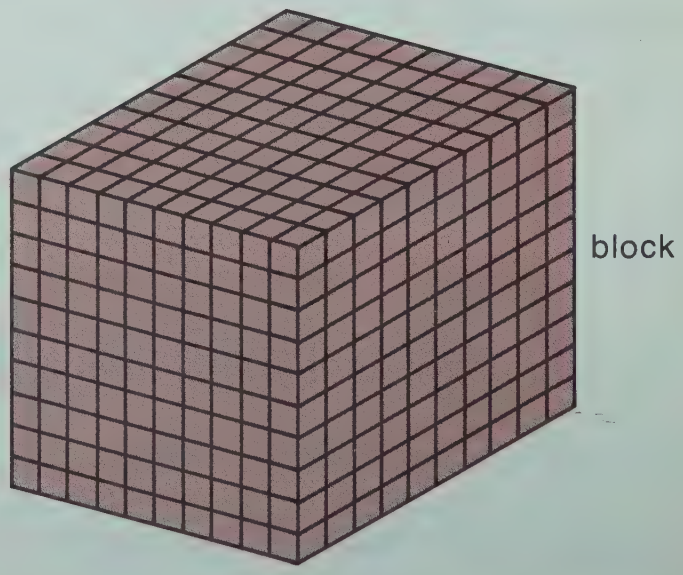
   B A rod is $\frac{1}{10}$ of a layer.

   C A layer is $\frac{1}{10}$ of a block.

   D A cube is $\frac{1}{100}$ of a layer.

   E A rod is $\frac{1}{100}$ of a block.

   F A cube is $\frac{1}{1000}$ of a block.

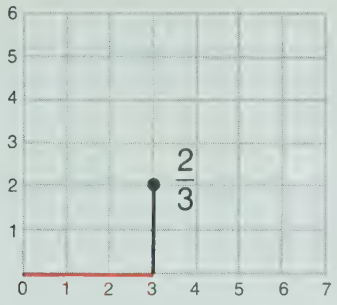


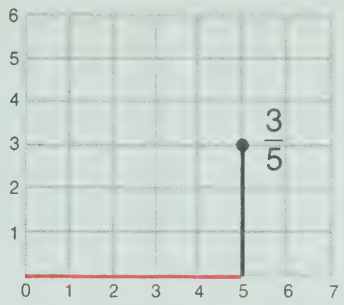
The piece of string in exercise 1D is to be one millimeter long. Although it will be very difficult to cut a piece of string this long, it should help children gain a feeling for the relationship between a millimeter and a meter.

Graphing Fractions

Study the examples. Then answer the questions.

EXAMPLES:

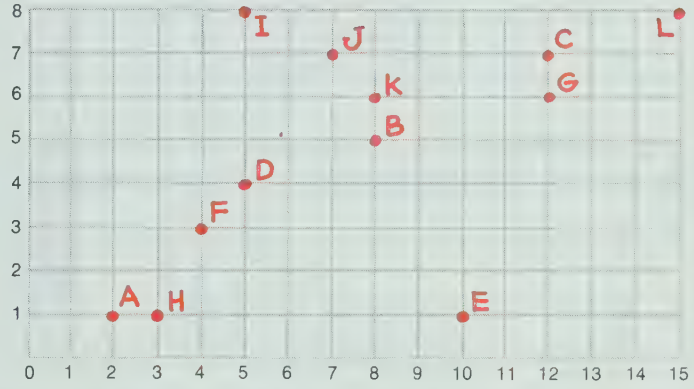




The dot is the graph of $\frac{2}{3}$ This dot is the graph of $\frac{3}{5}$

1. Graph these fractions and label them using the letters.
You need not draw the red and black lines used in the examples.

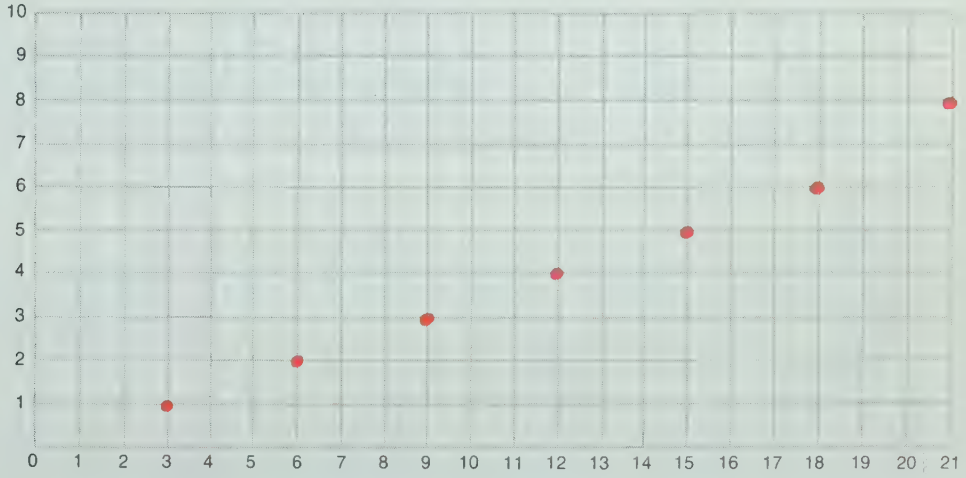
A $\frac{1}{2}$	E $\frac{1}{10}$	I $\frac{8}{5}$
B $\frac{5}{8}$	F $\frac{3}{4}$	J $\frac{7}{7}$
C $\frac{7}{12}$	G $\frac{6}{12}$	K $\frac{6}{8}$
D $\frac{4}{5}$	H $\frac{1}{3}$	L $\frac{8}{15}$



2. Graph the first 7 fractions in this set of equivalent fractions.

$\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12} \ldots\right\}$

$\frac{5}{15}, \frac{6}{18}, \frac{7}{21}$



A What did you discover about the dots? They lie on the same line

B Graph other sets of equivalent fractions on your graph paper.

Is your discovery still true? Yes

Multiple Strips and Equivalent Fractions

These strips are called **Multiple Strips** because each strip contains multiples of the first number on the strip.

1	2	3	4	5	6	7	8	9	10

4	8	12	16	20	24	28	32	36	40

2	4	6	8	10	12	14	16	18	20

5	10	15	20	25	30	35	40	45	50

3	6	9	12	15	18	21	24	27	30

10	20	30	40	50	60	70	80	90	100

1. Complete these two multiple strips for any numbers you choose. answers will vary.

A

B

2. When two multiple strips are placed together, are equivalent fractions formed? Yes

Complete these multiple strips to check this.

2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30

3. Make a set of equivalent fractions for each pair of blank strips below. Use only the multiple strips shown above. Choose only those in which the greatest common factor (GCF) of the numerator and denonimator of the first fraction is 1. First fractions: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}, \frac{2}{3}, \frac{2}{5}, \frac{3}{4}, \frac{3}{5}, \frac{3}{10}, \frac{4}{5}$, or their reciprocals.

A

B

C

D

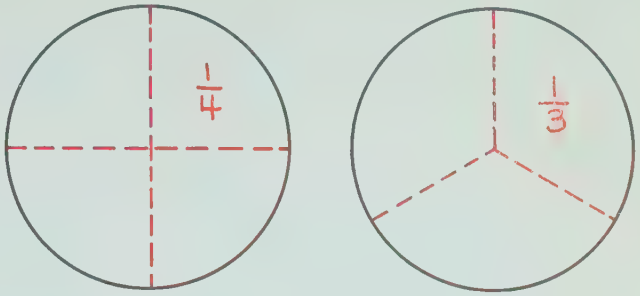
E

F

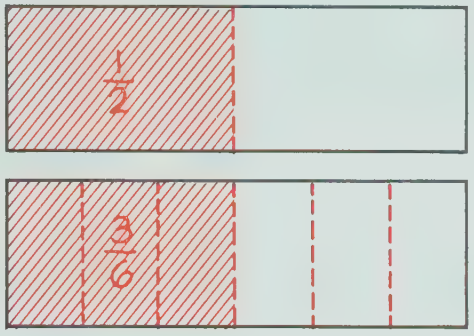
The important point to be made in this lesson is that by placing two of the multiple strips together a set of equivalent fractions is formed. Once children recognize this it will be much easier for them to construct sets of equivalent fractions.

Correcting Fraction Errors

1. Sara asked Eric if he wanted $\frac{1}{3}$ or $\frac{1}{4}$ of a pizza pie. Eric said "I'll take $\frac{1}{4}$! Everyone knows that $\frac{1}{4}$ is more than $\frac{1}{3}$." Draw pictures to show Eric his mistake.



2. Sue insisted that $\frac{1}{2}$ and $\frac{3}{6}$ could not be equivalent fractions because they have different numbers. Draw pictures to convince Sue that $\frac{1}{2}$ **is** equivalent to $\frac{3}{6}$.



3. Janelle tried to form a set of equivalent fractions. She wrote:

$$\left\{\frac{2}{3}, \frac{3}{4}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}\right\}$$

A Can you find the "wrong fraction" in the set?

B Make your own set of "equivalent fractions" and put a "wrong fraction" in the set. See if a classmate can find the "wrong fraction."

4. Fred said that he had discovered an easy way to reduce certain types of fractions that "always works." He wrote:

$$\frac{16}{64} = \frac{1}{4} \qquad \frac{19}{95} = \frac{1}{5}$$

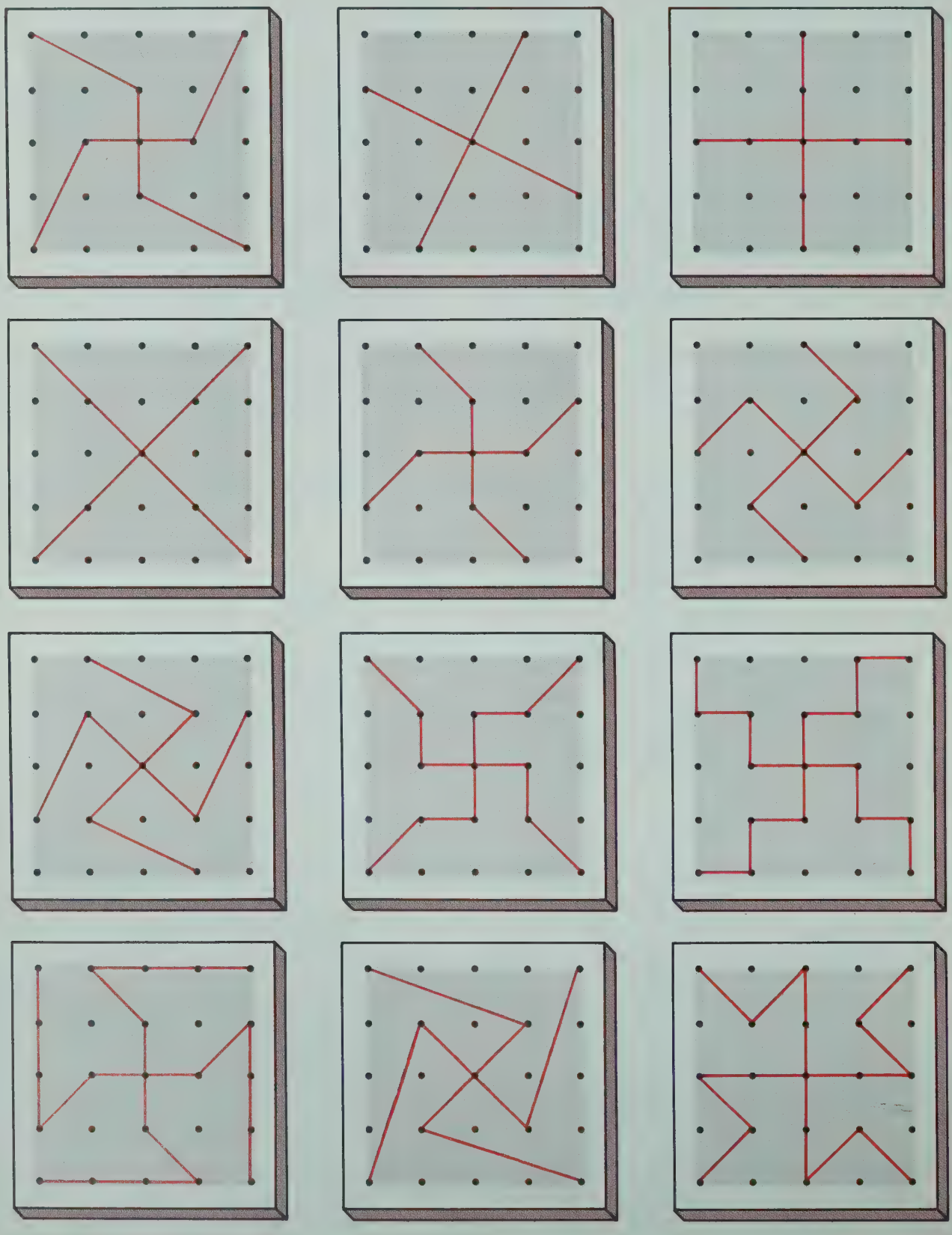
(Check these. Do they work?)

Give some fractions of this type that "don't work" to show Fred his method seldom works. Answers will vary

It is not necessary that children draw accurate pictures in exercises 1 and 2. The important thing is that their pictures represent the correct idea.

Fractions on the Geoboard

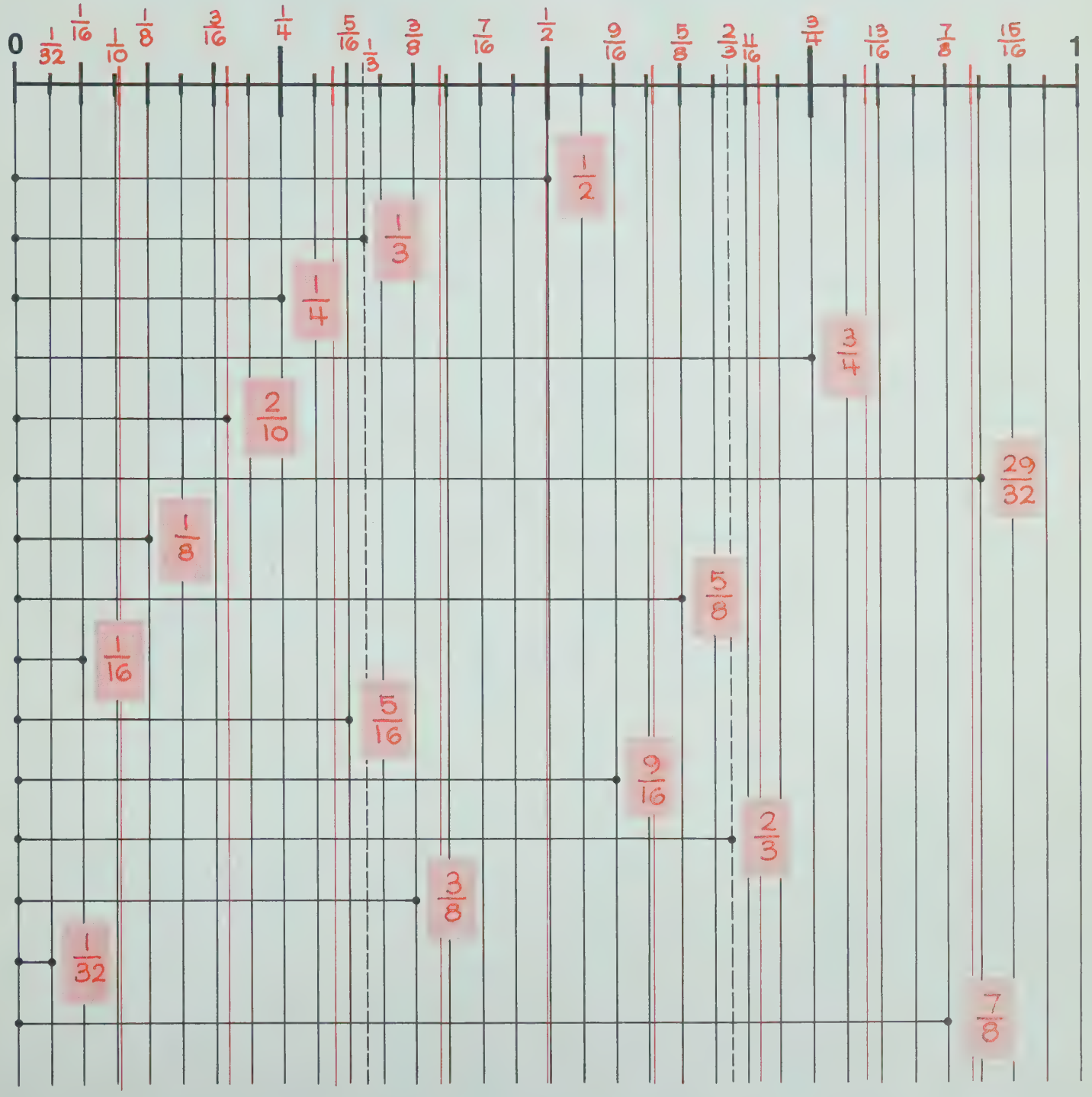
The first two pictures show ways to "divide a geoboard" in fourths in such a way that each of the 4 parts is the same size and shape. Show as many other different ways as you can to do this. Sample answers



It will be helpful if children have geoboards or sheets of dot paper in working with this page.

Fractional Number Lengths

Use the markings on the heavy "guide line" to help you write a fractional number which gives the length of each segment below. Label the "guide line" for easy reference.



It will be helpful if the children find the length of the various segments starting at the top and working down the page.

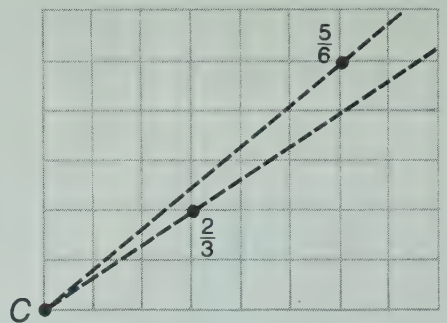
Another Way to Compare Fractional Numbers

1. Study this example:

- **A** Graph $\frac{2}{3}$
- **B** Graph $\frac{5}{6}$

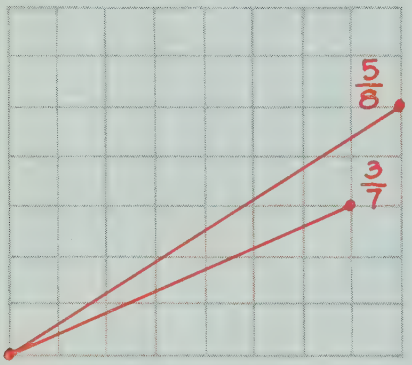
(See page 55 for a reminder on how to graph a fraction)

- **C** Draw lines from the corner point *C* through these dots.
- **D** Which line shows the "steepest hill?" $\frac{5}{6}$
- **E** Write $>$ or $<$ to indicate that the fraction with the "steepest hill" shows the larger number.

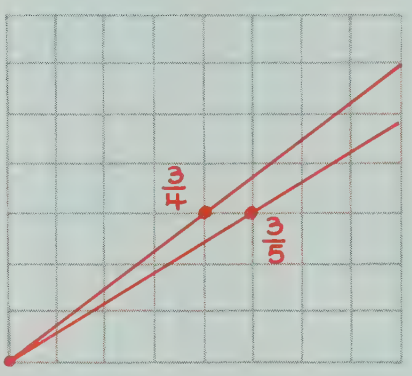


$\frac{2}{3} < \frac{5}{6}$

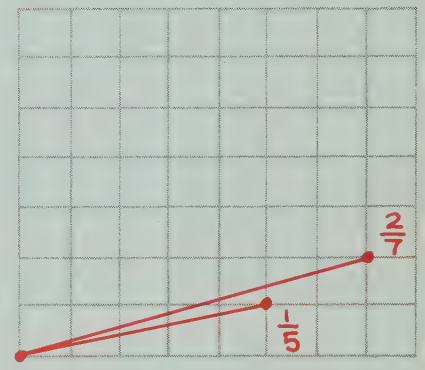
2. Draw the graphs and lines. Then write $<$ or $>$ in each



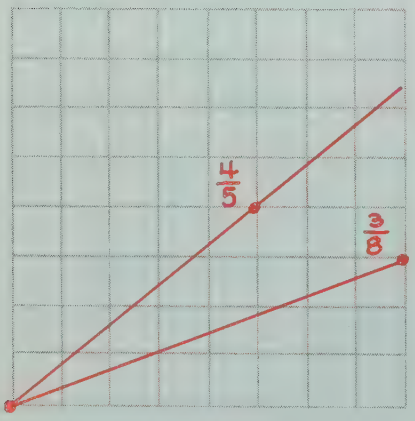
$\frac{3}{7} < \frac{5}{8}$



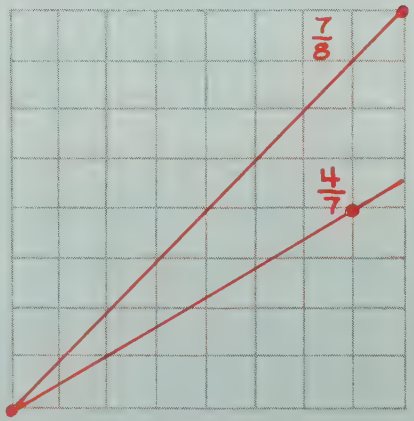
$\frac{3}{4} > \frac{3}{5}$



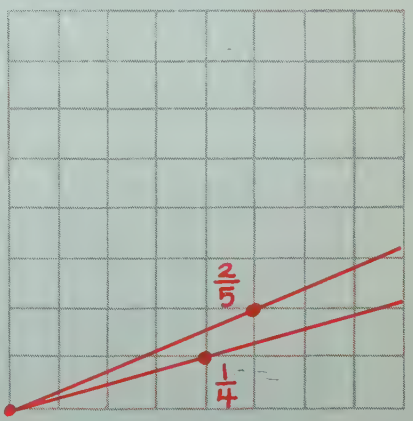
$\frac{1}{5} < \frac{2}{7}$



$\frac{4}{5} > \frac{3}{8}$



$\frac{4}{7} < \frac{7}{8}$



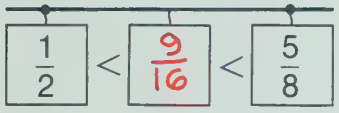
$\frac{2}{5} > \frac{1}{4}$

Between You and Me

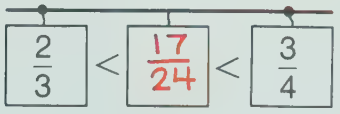
Find a fractional number that is between each of the two numbers given on the number line. Write it in the ☐

(Hint: Rewrite the two given fractions using equivalent fractions with the same denominators.) Sample answers

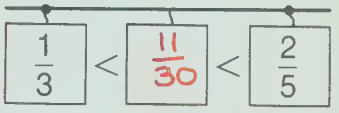
1.



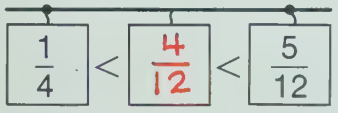
2.



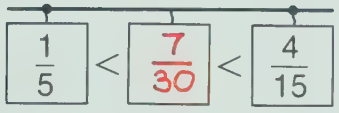
3.



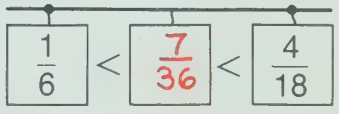
4.



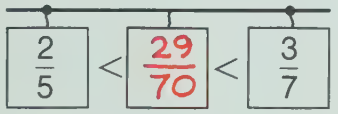
5.



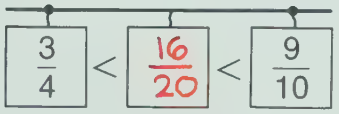
6.



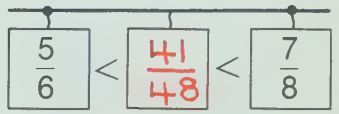
7.



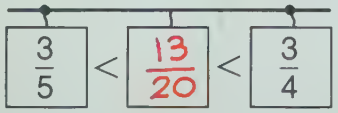
8.



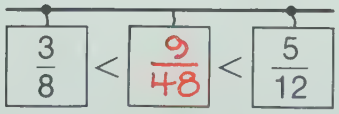
9.



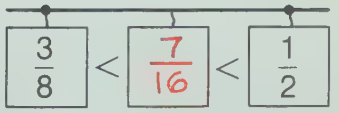
10.



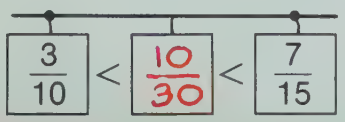
11.



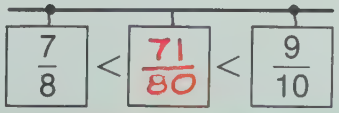
12.



13.



14.

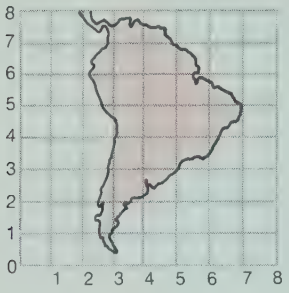


15.

It might be helpful to have the children check their answers to these exercises by one of the methods they have learned for determining which of two fractional numbers is greater.

Using Ratios to Enlarge Pictures

Make an enlargement of the small grid picture on the large grid. The numbers along the sides of the grid should help you locate matching points. Then give the **ratio** of distance on the large grid to the matching distance on the small grid.

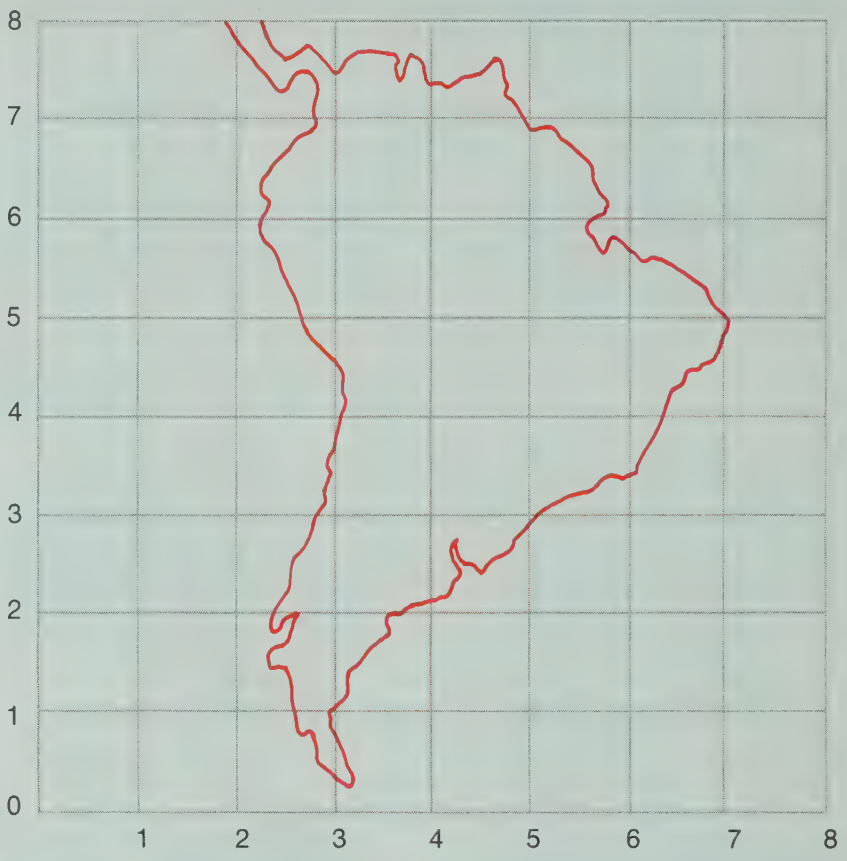


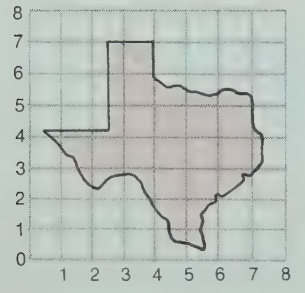
Small grid unit ⟷

Large grid unit ⟷

Ratio of large grid distance to small grid distance:

_____ to _____



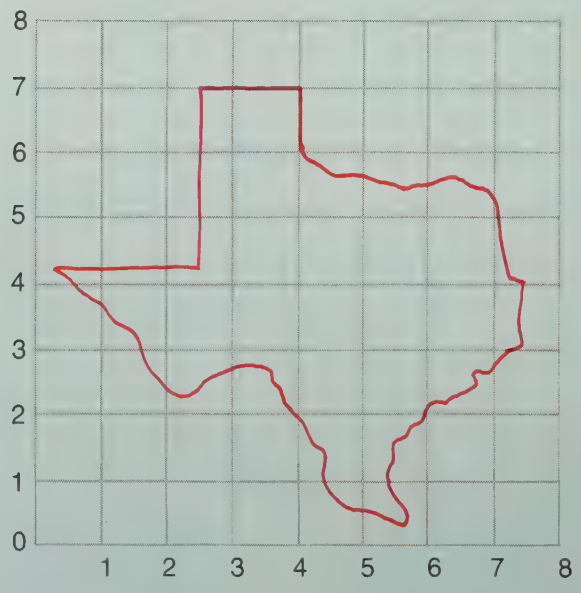


Small grid unit ⟷

Large grid unit ⟷

Ratio of large grid distance to small grid distance:

_____ to _____

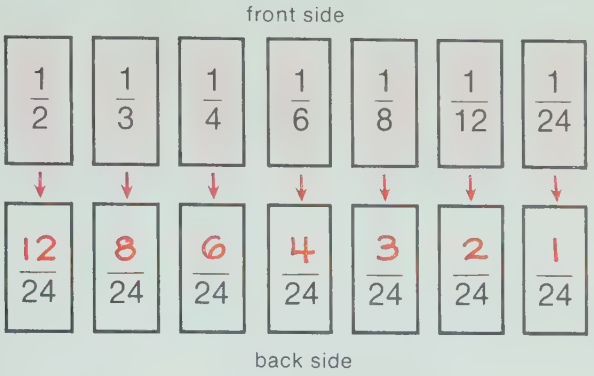


It will be helpful if children begin by marking specific points on the small grid before they attempt to find matching points on the larger grid.

Fun with Unit Fractions

Make or imagine a set of cards that have these **unit fractions** on the front.

Complete the cards so they have fractions on the back with denominator 24 which are equivalent to the fractions on the front.



Use the cards to help you look for ways to complete the equations below. For each exercise write a different **unit fraction** on the cards. Answers may vary

1. $\frac{1}{8} + \frac{1}{12} + \frac{1}{24} = \frac{1}{4}$

2. $\frac{1}{6} + \frac{1}{8} + \frac{1}{24} = \frac{1}{3}$

3. $\frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{1}{2}$

4. $\frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{3}{4}$

5. $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$

6. $\frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{12} + \frac{1}{24} = 1$

7. $\frac{1}{2} - \frac{1}{3} - \frac{1}{6} = 0$

8. $\frac{1}{3} - \frac{1}{4} - \frac{1}{12} = 0$

9. $\frac{1}{4} - \frac{1}{6} - \frac{1}{12} = 0$

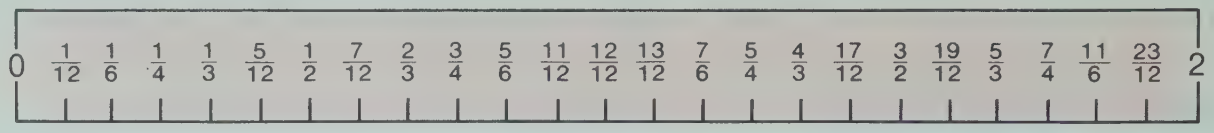
10. $\frac{1}{6} - \frac{1}{8} - \frac{1}{24} = 0$

11. $\frac{1}{2} + \frac{1}{8} + \frac{1}{24} = \frac{2}{3}$

Remind the children that thinking about each of the fractions involved in the addition or subtraction equations in terms of twenty-fourths, will help them find the different unit fractions involved.

A Fraction Slide Rule

Use tracing paper to make a copy of this "ruler." Put the division marks on the top instead of the bottom.



Use the two "rulers" as shown here to find the sum or difference of pairs of fractional numbers given below.

Choose 3 of the problems below and do them the usual way to check the slide rule answers.

$$\frac{1}{4}+\frac{1}{3}=\frac{7}{12} \quad \text{or} \quad \frac{7}{12}-\frac{1}{3}=\frac{1}{4}$$

1. $\frac{1}{4}+\frac{1}{12}=$ $\frac{1}{3}$
2. $\frac{7}{12}-\frac{1}{3}=$ $\frac{1}{4}$
3. $\frac{1}{6}+\frac{1}{4}=$ $\frac{5}{12}$
4. $\frac{5}{12}+\frac{1}{2}=$ $\frac{11}{12}$
5. $\frac{11}{12}-\frac{1}{3}=$ $\frac{7}{12}$
6. $\frac{1}{6}+\frac{2}{3}=$ $\frac{5}{6}$
7. $\frac{7}{12}+\frac{1}{4}=$ $\frac{5}{6}$
8. $\frac{3}{4}-\frac{5}{12}=$ $\frac{1}{3}$
9. $\frac{7}{12}+\frac{1}{3}=$ $\frac{11}{4}$
10. $\frac{5}{6}-\frac{5}{12}=$ $\frac{5}{12}$
11. $\frac{2}{3}+\frac{3}{4}=$ $\frac{17}{12}$
12. $\frac{3}{4}+\frac{5}{6}=$ $\frac{19}{12}$
13. $\frac{5}{3}-\frac{5}{6}=$ $\frac{5}{6}$
14. $2-\frac{7}{6}=$ $\frac{5}{6}$
15. $\frac{5}{6}+\frac{13}{12}=$ $\frac{23}{12}$

"Check" space

The check for the correct sums and differences will provide extra practice and should convince the children that the slide rule actually works.

Fraction Magic Squares

In a magic square, the sum of the numbers in each row, each column, and each main diagonal is the same. This sum is called the "magic number" of the square. Can you complete these magic squares?

Fractions may or may not be in lowest terms.

1.

$\frac{2}{3}$	$\frac{1}{12}$	$\frac{1}{2}$
$\frac{1}{4}$	$\frac{5}{12}$	$\frac{7}{12}$
$\frac{1}{3}$	$\frac{3}{4}$	$\frac{1}{6}$

Magic Number $\frac{15}{12}$ or $\frac{5}{4}$

2.

$\frac{1}{2}$	4	$\frac{3}{2}$
3	2	1
$\frac{5}{2}$	0	$\frac{7}{2}$

Magic Number 6

3.

2	$\frac{1}{4}$	$\frac{6}{4}$
$\frac{3}{4}$	$\frac{5}{4}$	$\frac{7}{4}$
1	$\frac{9}{4}$	$\frac{1}{2}$

Magic Number $\frac{15}{4}$

4.

$\frac{1}{2}$	$\frac{3}{8}$	1
$\frac{9}{8}$	$\frac{5}{8}$	$\frac{1}{8}$
$\frac{1}{4}$	$\frac{7}{8}$	$\frac{3}{4}$

Magic Number $\frac{15}{8}$

5.

$\frac{2}{5}$	$\frac{7}{5}$	$\frac{6}{5}$
$\frac{9}{5}$	1	$\frac{1}{5}$
$\frac{4}{5}$	$\frac{3}{5}$	$\frac{8}{5}$

Magic Number $\frac{15}{5}$ or 3

6.

$\frac{7}{12}$	0	$\frac{5}{12}$
$\frac{2}{12}$	$\frac{4}{12}$	$\frac{6}{12}$
$\frac{1}{4}$	$\frac{8}{12}$	$\frac{1}{12}$

Magic Number 1

7.

$\frac{13}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{16}{4}$
$\frac{8}{4}$	$\frac{11}{4}$	$\frac{5}{2}$	$\frac{5}{4}$
$\frac{12}{4}$	$\frac{7}{4}$	$\frac{3}{2}$	$\frac{9}{4}$
$\frac{1}{4}$	$\frac{7}{2}$	$\frac{15}{4}$	1

Magic Number $\frac{17}{2}$

8.

$\frac{7}{4}$	$\frac{12}{4}$	$\frac{9}{4}$	$\frac{3}{2}$
$\frac{7}{2}$	$\frac{1}{4}$	1	$\frac{15}{4}$
$\frac{2}{4}$	$\frac{13}{4}$	$\frac{16}{4}$	$\frac{3}{4}$
$\frac{11}{4}$	$\frac{8}{4}$	$\frac{5}{4}$	$\frac{5}{2}$

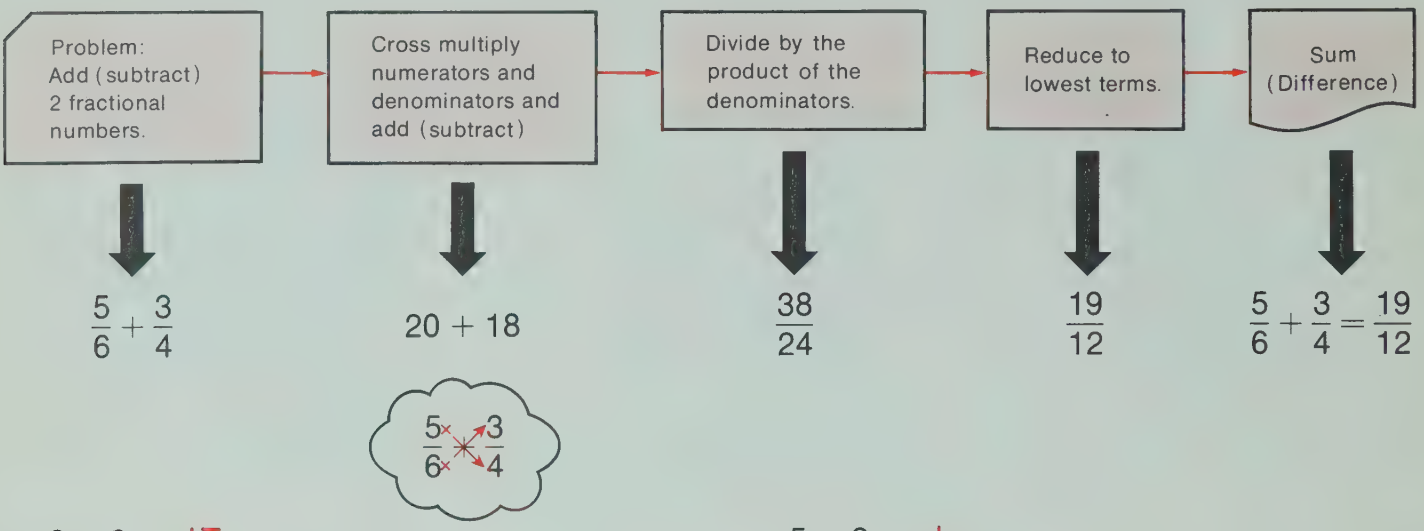
Magic Number $\frac{34}{4}$ or $\frac{17}{2}$

Remind the children that in working out the various fractions it will be helpful to have all the fractions with the same denominator. When they give their answers they should reduce all of the fractions to lowest terms.

A Fractional Number Shortcut

This flow chart shows a "shortcut" for adding or subtracting fractional numbers. Study the example and use it to find the sums and differences below.

Check two of the problems to see if the shortcut worked.



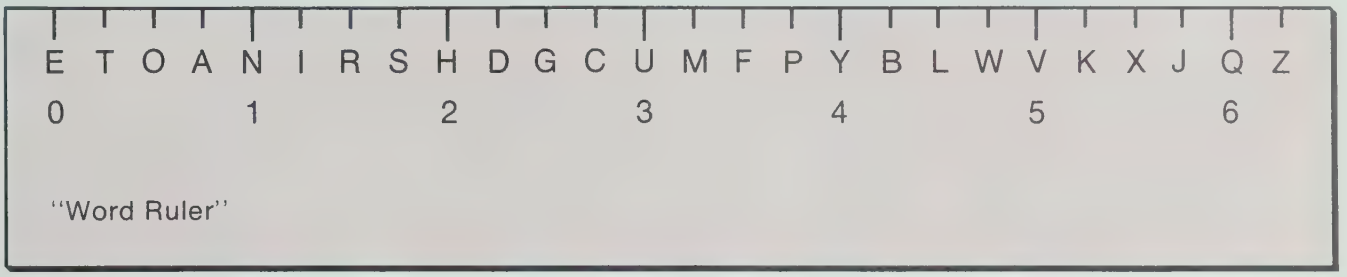
1. $\frac{2}{3}+\frac{3}{4}=\frac{17}{12}$
2. $\frac{3}{8}+\frac{4}{7}=\frac{53}{56}$
3. $\frac{5}{6}+\frac{7}{8}=\frac{41}{24}$
4. $\frac{3}{5}+\frac{2}{3}=\frac{19}{15}$
5. $\frac{5}{8}+\frac{7}{12}=\frac{29}{24}$
6. $\frac{1}{6}+\frac{7}{8}=\frac{25}{24}$
7. $\frac{5}{6}-\frac{3}{4}=\frac{1}{12}$
8. $\frac{7}{8}-\frac{2}{7}=\frac{33}{56}$
9. $\frac{2}{3}-\frac{3}{5}=\frac{1}{15}$
10. $\frac{1}{2}-\frac{3}{8}=\frac{1}{8}$
11. $\frac{3}{4}-\frac{2}{5}=\frac{7}{20}$
12. $\frac{7}{8}-\frac{7}{12}=\frac{7}{24}$

"Check" space

Note that in some cases this method for finding the sum of two fractional numbers gives the answer in lowest terms. In others, the answer must be reduced to lowest terms.

Measure Your Words!

Here is a special ruler you can use to find the "length" of a word. Study the examples and answer the questions.



"Word Ruler"

EXAMPLES: F U N
$3\frac{1}{2} + 3 + 1 = 7\frac{1}{2}$

M A T H
$3\frac{1}{4} + \frac{3}{4} + \frac{1}{4} + 2 = 6\frac{1}{4}$

"Fun" is a "longer word" than "Math."

1. Can you find the "length" of your name? _____

2. Which is "longer," your first name or your last name? _____

3. Give the length of each word.

A CUT $2\frac{3}{4}+3+\frac{1}{4}=6$ B DOT $2\frac{1}{4}+\frac{1}{2}+\frac{1}{4}=3$ C BIT $4\frac{1}{4}+1\frac{1}{4}+\frac{1}{4}=5\frac{3}{4}$

D ROB $1\frac{1}{2}+\frac{1}{2}+4\frac{1}{4}=6\frac{1}{4}$ E JET $5\frac{3}{4}+0+\frac{1}{4}=6$ F WIN $4\frac{3}{4}+1\frac{1}{4}+\frac{1}{4}=6\frac{1}{4}$

4. Each sum below represents a name. Find the sum and the name.

J A C K
$5\frac{3}{4} + \frac{3}{4} + 2\frac{3}{4} + 5\frac{1}{4} = 14\frac{1}{2}$

S A L L Y
$1\frac{3}{4} + \frac{3}{4} + 4\frac{1}{2} + 4\frac{1}{2} + 4 = 15\frac{1}{2}$

5. Can you find a word with length 6? answers will vary

No significance is attached to the number assigned to any given letter. This lesson is designed primarily to give children practice in adding fractional numbers.

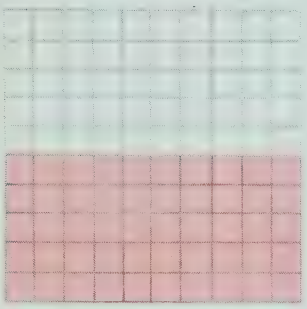
Cracking the Code

Use the Word Ruler on page 67 to decode this question. (Read down the column.)
The answer to the question is on the bottom of page 70.

	Answer	Letter
1. $2\frac{7}{8} + 1\frac{7}{8} =$	$4\frac{3}{4}$	W
2. $7\frac{1}{4} - 5\frac{3}{12} =$	2	H
3. $6\frac{7}{12} - 5\frac{5}{6} =$	$\frac{3}{4}$	A
4. $4\frac{1}{8} - 3\frac{7}{8} =$	$\frac{1}{4}$	T
5. $1\frac{7}{16} + \frac{13}{16} =$	$2\frac{1}{4}$	D
6. $6\frac{1}{8} - 4\frac{7}{8} =$	$1\frac{1}{4}$	I
7. $1\frac{3}{7} + \frac{23}{28} =$	$2\frac{1}{4}$	D
8. $1\frac{1}{8} - \frac{7}{8} =$	$\frac{1}{4}$	T
9. $5\frac{7}{8} - 3\frac{7}{8} =$	2	H
10. $8\frac{1}{2} - 8\frac{1}{2} =$	0	E
11. $8\frac{7}{12} - 3\frac{5}{6} =$	$4\frac{3}{4}$	W
12. $37\frac{1}{16} - 35\frac{13}{16} =$	$1\frac{1}{4}$	I
13. $\frac{7}{8} + \frac{7}{8} =$	$1\frac{3}{4}$	S
14. $\frac{0}{3} - \frac{0}{5} =$	0	E
15. $2\frac{5}{7} + 1\frac{11}{14} =$	$4\frac{1}{2}$	L
16. $9\frac{1}{5} - 7\frac{19}{20} =$	$1\frac{1}{4}$	I
17. $\frac{1}{12} + \frac{1}{6} =$	$\frac{1}{4}$	T
18. $18\frac{1}{16} - 17\frac{13}{16} =$	$\frac{1}{4}$	T
19. $3\frac{1}{8} + 1\frac{3}{8} =$	$4\frac{1}{2}$	L
20. $6\frac{1}{2} - 6\frac{5}{10} =$	0	E
21. $\frac{9}{10} + \frac{17}{20} =$	$1\frac{3}{4}$	S
22. $2\frac{1}{3} + \frac{17}{12} =$	$3\frac{3}{4}$	P
23. $10 - 8\frac{5}{10} =$	$1\frac{1}{2}$	R
24. $8 - 7\frac{9}{18} =$	$\frac{1}{2}$	O
25. $1\frac{5}{8} + 1\frac{3}{8} =$	3	U
26. $12\frac{1}{5} - 11\frac{19}{20} =$	$\frac{1}{4}$	T
27. $\frac{9}{16} + \frac{19}{16} =$	$1\frac{3}{4}$	S
28. $6\frac{5}{9} - 5\frac{29}{36} =$	$\frac{3}{4}$	A
29. $2\frac{1}{8} + 1\frac{21}{24} =$	4	Y
		?

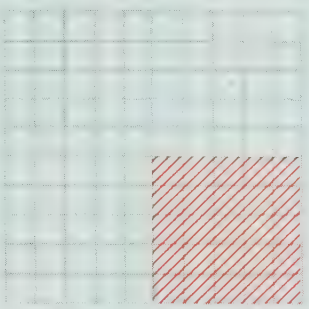
Color the part of the grid indicated. Write the missing fraction, decimal, or percent.

EXAMPLE:



fraction $\frac{1}{2}$
decimal .50
percent 50 %
} of the grid is shaded.

1.

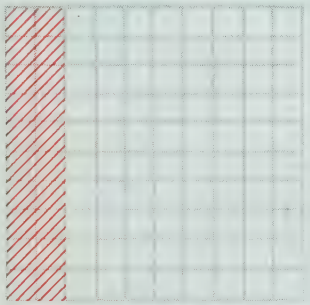


fraction $\frac{1}{4}$
decimal .25
percent 25 %
} of the grid is shaded.

2.

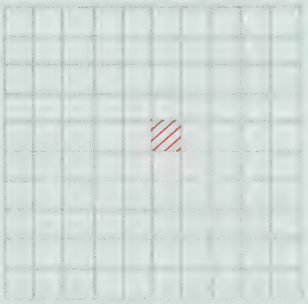
fraction $\frac{3}{4}$
decimal .75
percent 75 %
} of the grid is shaded.

3.



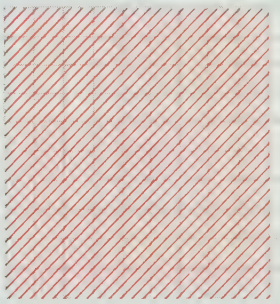
fraction $\frac{1}{5}$ decimal .20
percent 20 %

4.



fraction $\frac{1}{100}$ decimal .01
percent 1 %

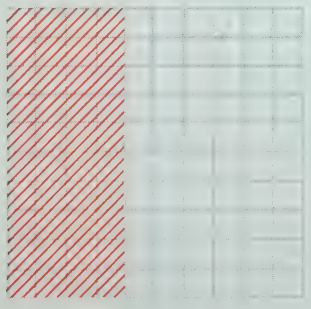
5.



fraction $\frac{9}{10}$ decimal .90
percent 90 %

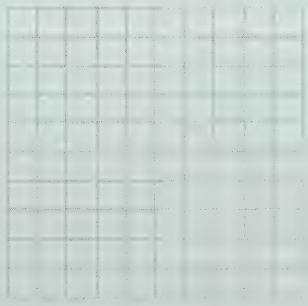
_____ choose your own _____

6.



fraction $\frac{4}{10}$ decimal .40
percent 40 %

7.



fraction _____ decimal _____
percent _____ %

8.

fraction _____ decimal _____
percent _____ %

Answers will vary

Since the emphasis on this page is upon decimals and percents be sure the children recognize that each grid has 100 small squares. This will not only help them decide on how many squares to shade but also strengthen their understanding of decimals and percents.

A Graphical Look at Decimals and Fractions

Graph as many fractions from each set of equivalent fractions as you can and draw a line for this set on the graph. Use the decimal scale to give or estimate, the decimal for each set.

EXAMPLE: $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\right\}$ .50

$\left\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \ldots\right\}$ .10

$\left\{\frac{3}{8}, \frac{6}{16}, \frac{9}{24}, \frac{12}{32}, \ldots\right\}$ .38

$\left\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \ldots\right\}$ .83

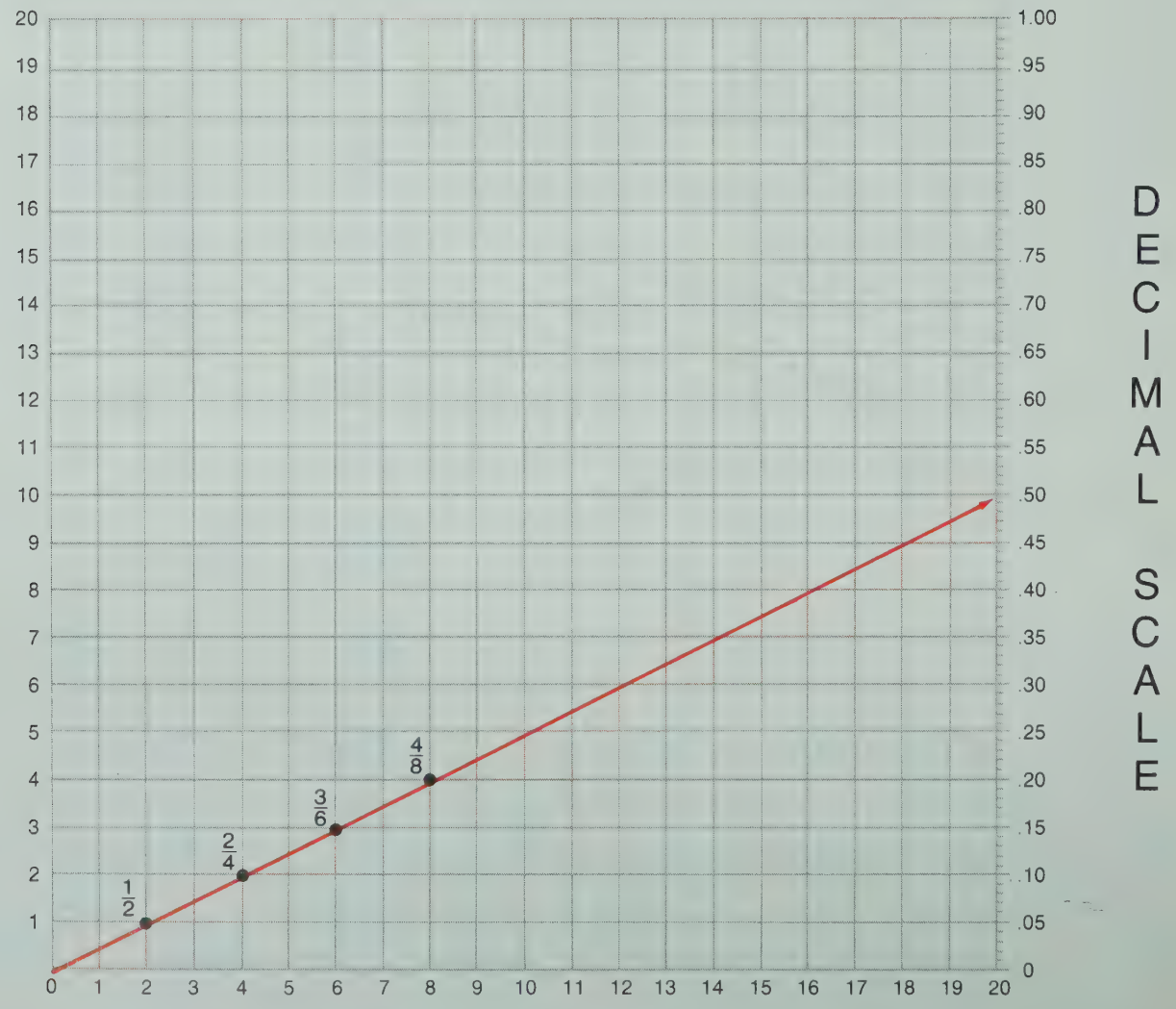
$\left\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\right\}$ .16

$\left\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \ldots\right\}$ .13

$\left\{\frac{7}{8}, \frac{14}{16}, \frac{21}{24}, \frac{28}{32}, \ldots\right\}$ .88

$\left\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\right\}$ .33

Lines should intersect the decimal scale at the given points.



"Geometry" say it slowly

Writing Story Problems Using Decimals

Use the information given to write a story problem using decimals which you can solve. Show your solution. Answers will vary

1.

Average cost of electricity about 3.56¢ for each kilowatt hour.	Kilowatt hours used per month. Toaster-about 3 Mixer-about 1 Sink disposal-about 2 Radio-about 10

2.

Gas Record Wednesday purchase 30 liters super Thursday purchase 42 liters regular	Super gasoline $.14 per liter Regular gasoline $.13 per liter

3.

Car 9.5 kilometers per liter Motorcycle 14.5 kilometers per liter

Gasoline Used 8 liters

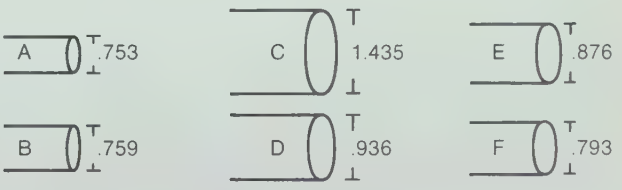
4.

Daily Expenses Mon: $5.75 Tues: $3.82 Wed: $4.97 Thurs: $6.39 Fri: $7.45

5.

Cereal A		Cereal B	
Vitamin B_1	.17 mg	Vitamin B_1	.23 mg
Niacin	1.1 mg	Niacin	4.25 mg
Iron	22.5 mg	Iron	3.46 mg
Phosphorus	87.6 mg	Phosphorus	123.67 mg
Riboflavin	.8 mg	Riboflavin	.15 mg

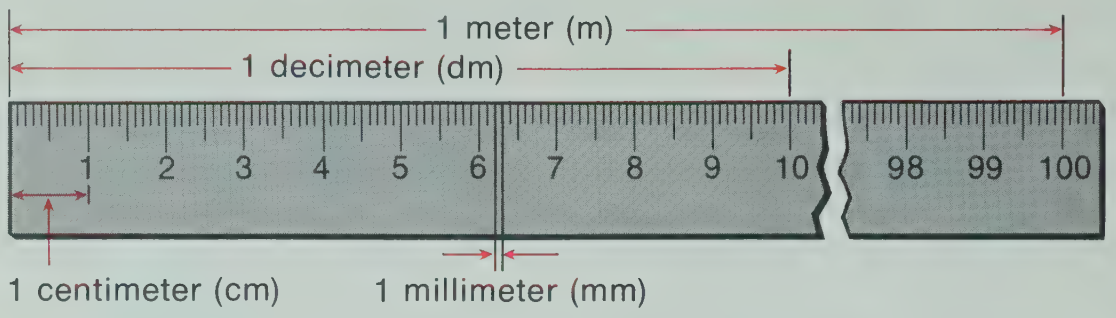
6.



Diameter of steel rods

Decimals and Metric Measurement

1. Complete this chart to help you understand the relationships between some of the units of length in the metric system. (Use decimals.)

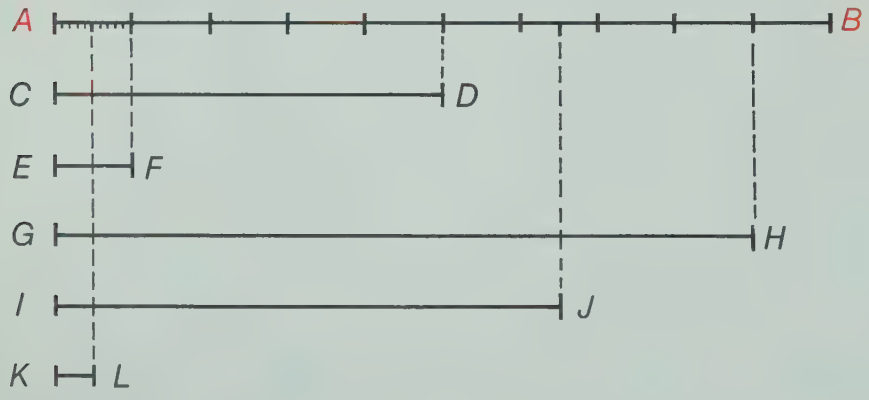


There are 10 decimeters in a meter. A decimeter is .10 of a meter.

There are 10 centimeters in a decimeter. A centimeter is .10 of a decimeter.

There are 100 centimeters in a meter. A centimeter is .01 of a meter.

Segment *AB* has length 1 decimeter. .10 of segment *AB* has length 1 centimeter. .01 of segment *AB* has length 1 millimeter.

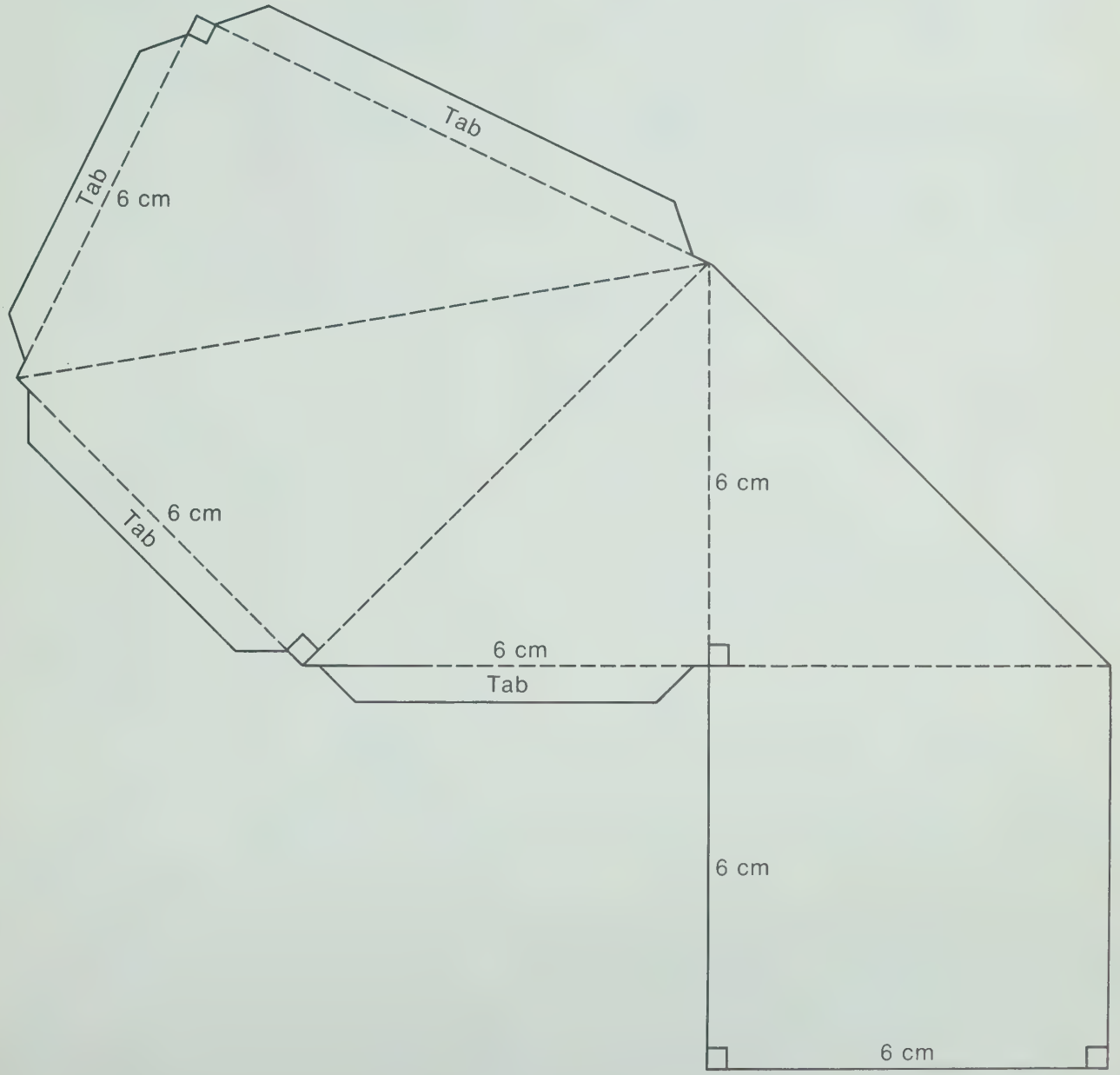


2. Write decimals or whole numbers to complete this chart.

Segment	Length in millimeters	Length in centimeters	Length in decimeters	Length in meters
$\overline{AB}$	100	10	1	.10
$\overline{CD}$	50	5	.50	.05
$\overline{EF}$	10	1	.10	.01
$\overline{GH}$	90	9	.90	.09
$\overline{IJ}$	65	6.5	.65	.065
$\overline{KL}$	5	.50	.05	.005

A Solid Puzzle

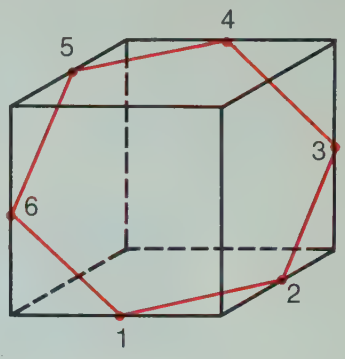
Draw 3 of these patterns on light poster board and cut them out. Fold on the dotted lines and tape the edges together to make pyramids.  Can you place the 3 pyramids together to form a cube?



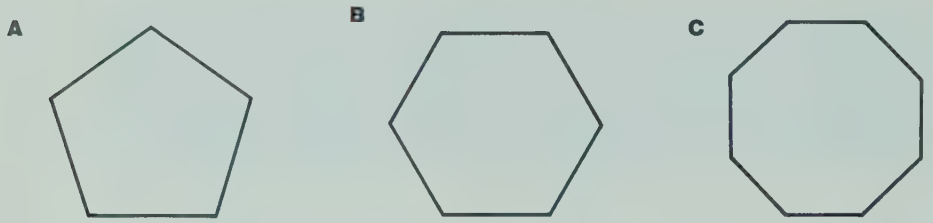
Three pyramids of this shape can be placed together to form a cube. Encourage the children to keep trying until they have solved the puzzle.

Cube Capers

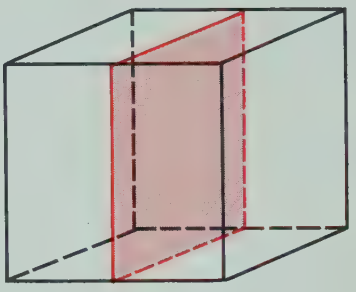
Can you answer these questions about a cube?

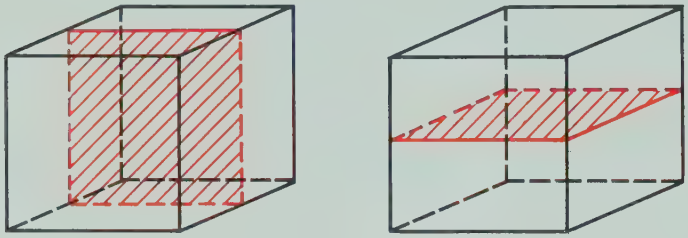


1. If you connect the points the resulting figure would look like B

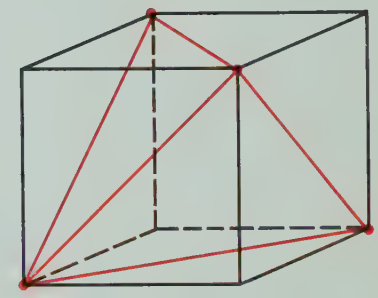


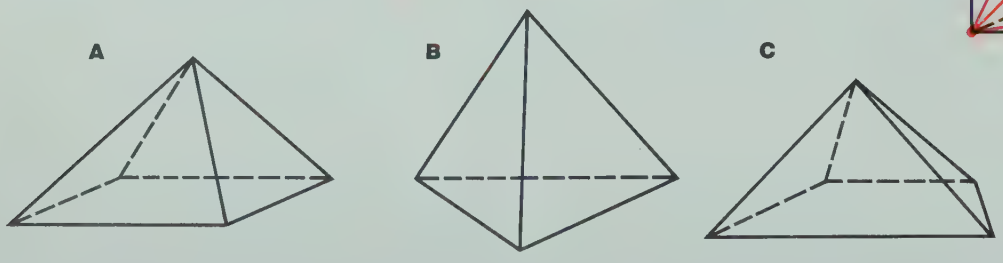
2. The red plane is parallel to two faces of the cube and divides the cube into two identical parts. Draw two more planes, different from this one, that do the same thing.



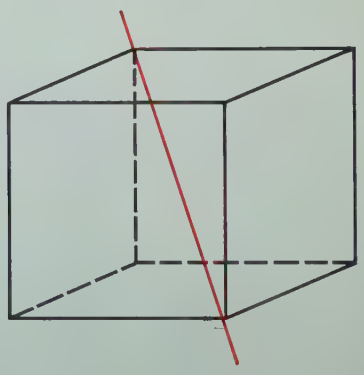


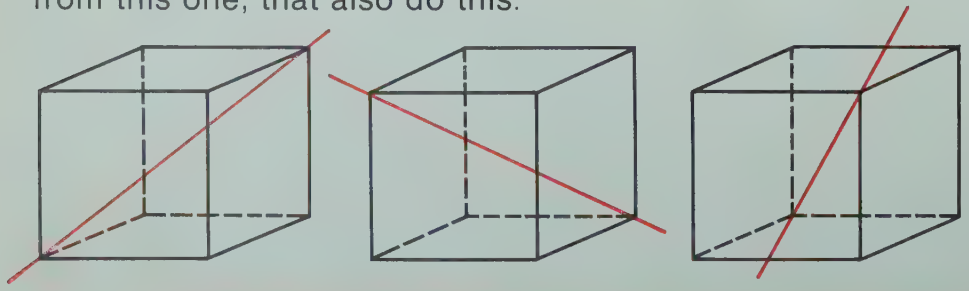
3. A space figure has been made by connecting 4 vertices of the cube with red lines. This figure, if removed and placed on a table, would look like B





4. The red line goes through opposite vertices of the cube. Show 3 red lines, in different positions from this one, that also do this.



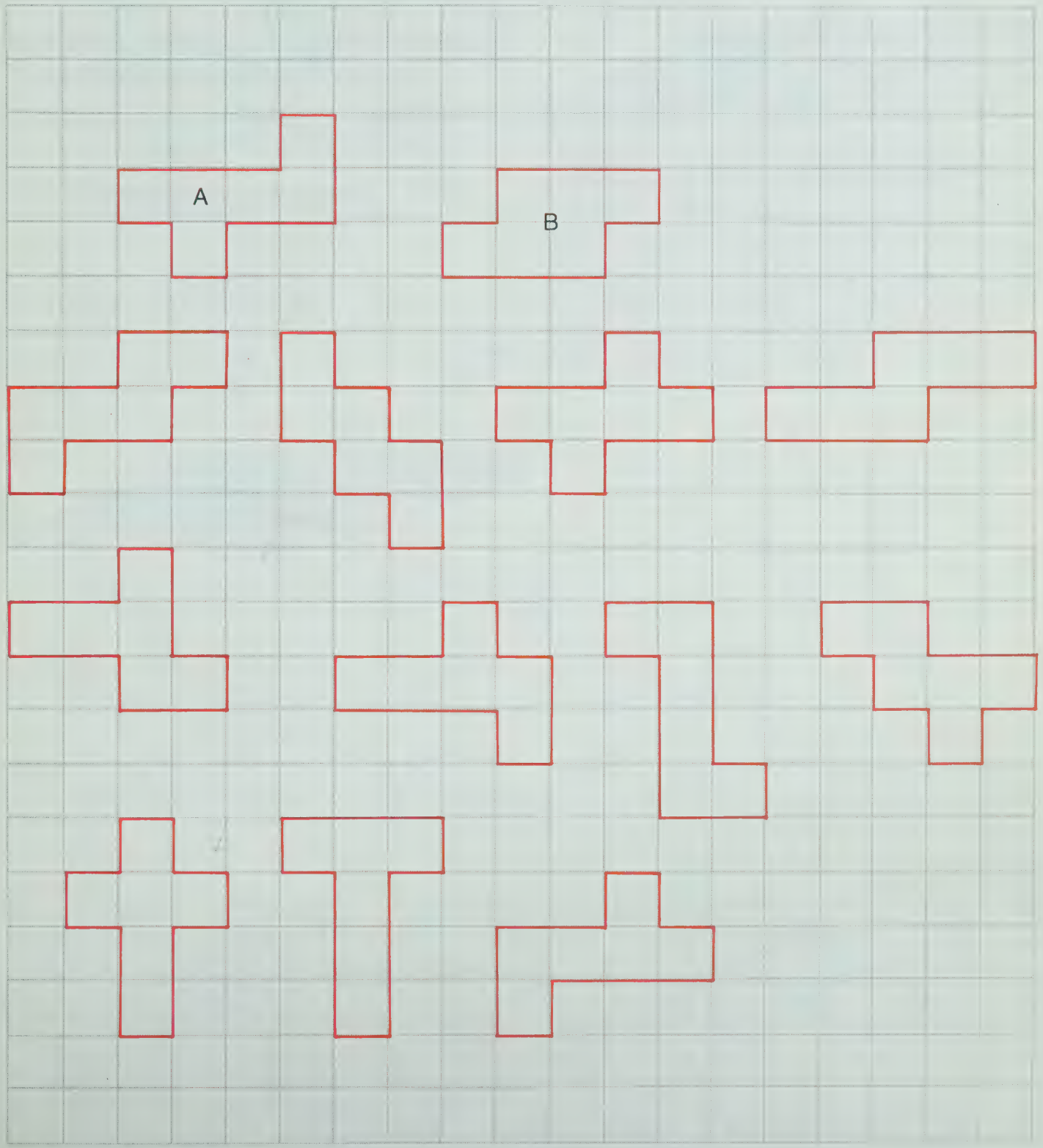


It will be helpful if the children actually have a cube to look at as they solve these exercises. Additional challenges can be found in having the children try to construct the figures formed for exercises 1 and 3.

Finding Patterns for Cubes

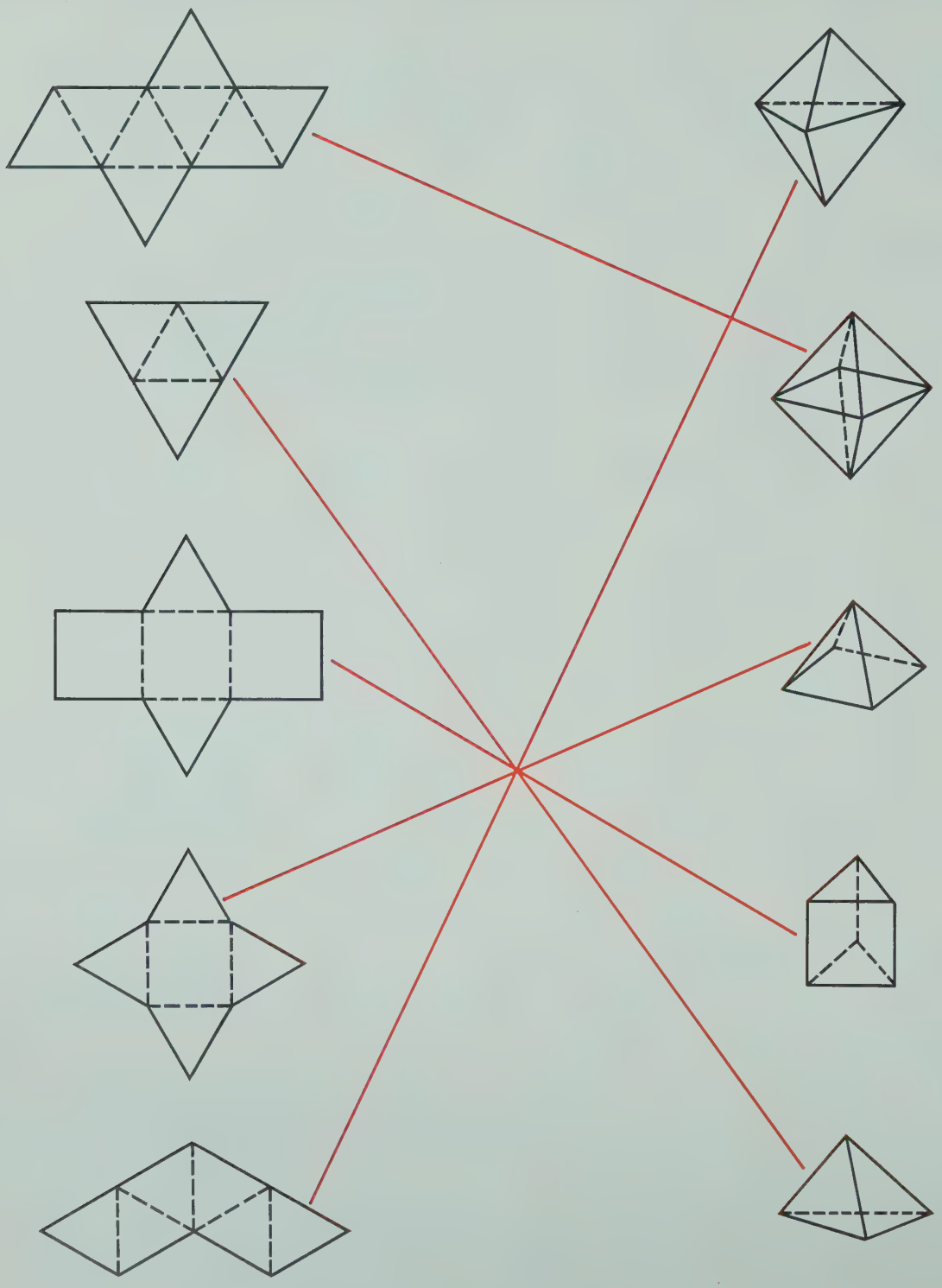
Pattern A can be folded to form a cube. Pattern B cannot. There are 11 patterns which, like A, can be folded to make a cube.

How many of them can you draw? (You may want to cut your patterns out and try them.)



Pattern Problems

Each pattern in the first column can be folded to make one of the space figures in the second column. Draw lines to match each pattern with its completed space figure.

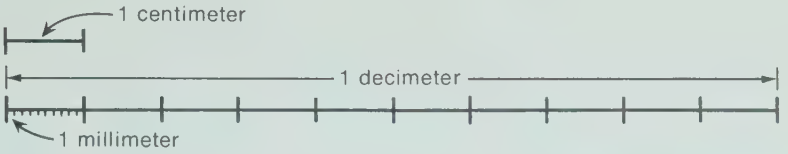


Use one of the patterns above and make one of the space figures.

Be sure the children attempt to do the matching of the plane and space figures prior to constructing any of the space figures.

Volume in the Metric System

Remember:
10 decimeters = 1 meter
10 centimeters = 1 decimeter
10 millimeters = 1 centimeter



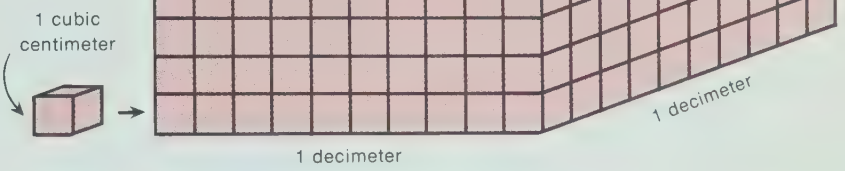
Can you answer these questions about volume in the metric system?

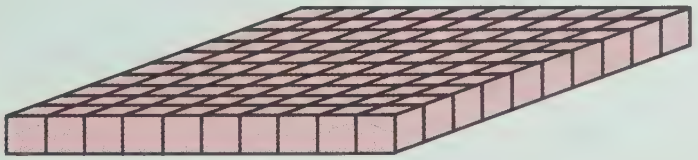
1. Suppose that this is a picture of a cubic decimeter and a cubic centimeter.

A How many cubic centimeters are in one "row" of the cubic decimeter?

10

B How many "rows" are in one "layer" of the cubic decimeter? 10





C How many layers are in the cubic decimeter? 10

D How many cubic centimeters are needed to form a cubic decimeter? 1000

The volume of 1 cubic decimeter = 1000 cubic centimeters.

2. **A** How many cubic decimeters are needed to make a "row" 1 meter long? 10

B How many such rows are needed to make a layer that is 1 meter on each side? 10

C How many such layers are needed to make a cubic meter? 10

D How many cubic decimeters are needed to make a cubic meter? 1000

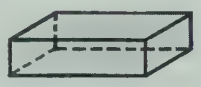
The volume of a cubic meter = 1000 cubic decimeters.

3. How many cubic millimeters are needed to make a cubic centimeter? 1000

a cubic decimeter? 1,000,000 a cubic meter? 1,000,000,000

Finding Volume and Surface Area

Use your ruler and measure "objects" in your classroom or at home that are shaped like rectangular prisms. Choose an appropriate unit (millimeter, centimeter, decimeter, or meter) and measure the length, width, and height of each object. Then find the volume and surface area of the "object."



Answers will vary

1. A chalkboard eraser 

Length _____ ..... your chosen unit

Width _____ .....

Height _____ .....

Volume = _____ ..... unit

Surface area = _____ ..... unit

2. A crayon or pencil box

Length _____ ..... your chosen unit

Width _____ .....

Height _____ .....

Volume = _____ ..... unit

Surface area = _____ ..... unit

3. Your Mathematics book 

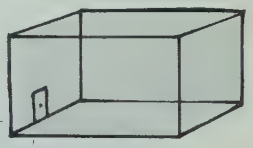
Length _____ ..... your chosen unit

Width _____ .....

Height _____ .....

Volume _____ ..... unit

Surface area _____ ..... unit

4. Your classroom 

Length _____ ..... your chosen unit

Width _____ .....

Height _____ .....

Volume _____ ..... unit

Surface area _____ ..... unit

5. A shoe box, gift box, or other box. 

Length _____ ..... your chosen unit

Width _____ .....

Height _____ .....

Volume _____ ..... unit

Surface area _____ ..... unit

6. An "object" of your choice

Length _____ ..... your chosen unit

Width _____ .....

Height _____ .....

Volume _____ ..... unit

Surface area _____ ..... unit

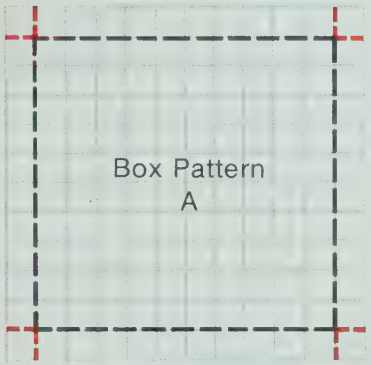
Exploring Volume and Surface Area

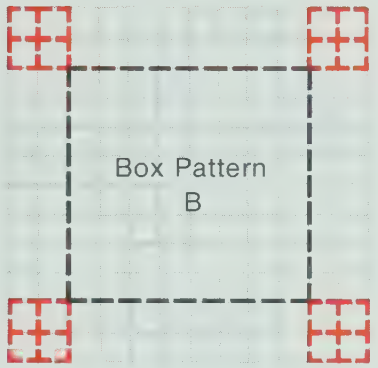
1. When does the volume of a cube equal the surface area of the cube?

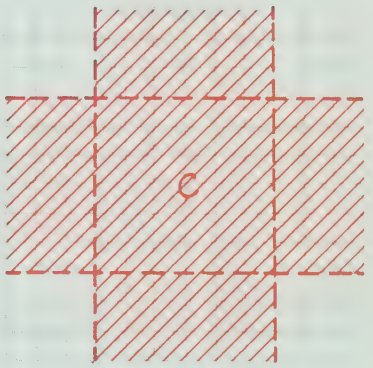
Complete as many lines of this table as necessary to answer the question.

When the length of the side of the cube is 6 units.

Length of side of cube	Volume	Surface area
1	1	6
2	8	24
3	27	54
4	64	96
5	125	150
6	216	216
7	343	294
8	512	384



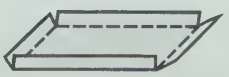


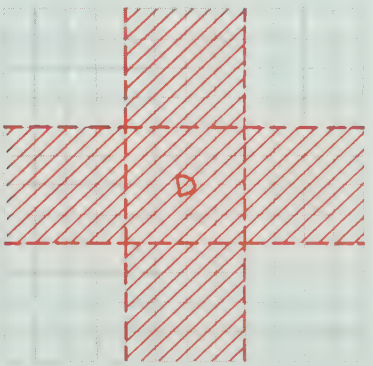


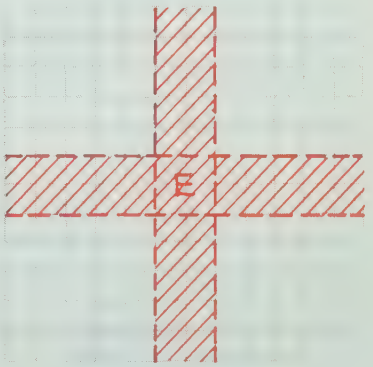
2. Start with a 12 by 12 piece of graph paper. Make patterns for boxes by first cutting 1 square from each corner, then 4, then 9, then 16, then 25. Guess which box has the

largest volume? _____

Surface area? _____
(without the top)





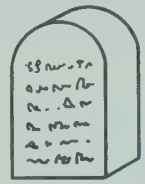
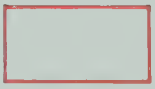
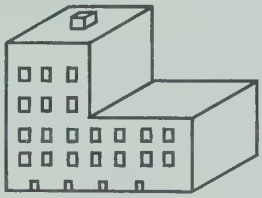
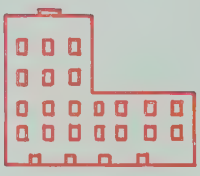
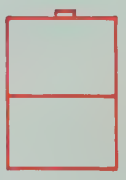
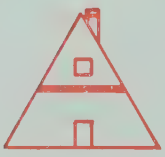
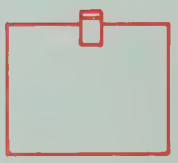
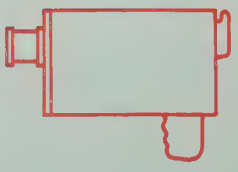
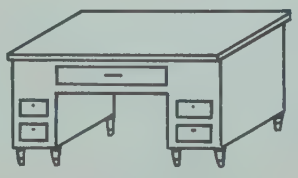
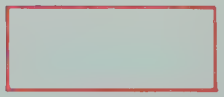


Complete this table to check your guesses.

Box	A	B	C	D	E
Volume	100	128	108	64	20
Surface Area (don't count the top of the box)	140	128	108	80	44

What's Your Point of View?

Can you draw the top view, front view, and side view for each object?

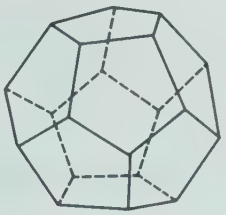
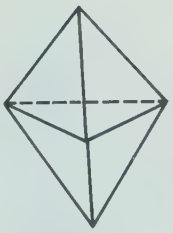
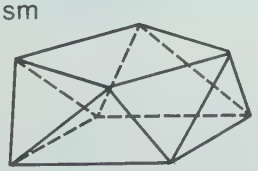
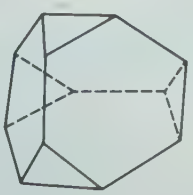
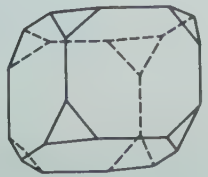
Object	Top View	Front View	Side View
 Historical marker stone			
 Equipment shed			
 Refrigerator-freezer			
 Office building			
 "A-Frame" house			
 Movie camera			
 Office desk			

If children have difficulty with this page it will be helpful for them to attempt to construct from clay, or other suitable material, the object pictured.

Checking a Formula

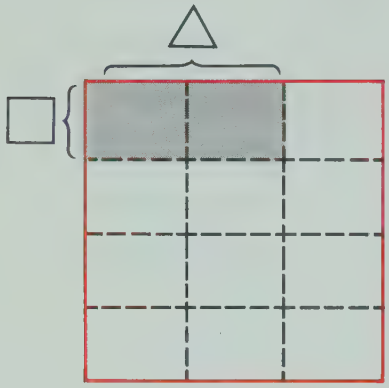
After exploring some simple solid figures, one class decided that "the number of vertices (V) + the number of faces (F) − 2 equals the number of edges (E)."

Check this formula for each solid figure below. Does it work? Yes

Solid figure	Number of vertices V	Number of faces F	V + F − 2	Number of Edges E
Tetrahedron equilateral triangle faces 	4	4	6	6
Dodecahedron each face is a pentagon 	20	12	30	30
Deltahedron all faces are equilateral triangles 	5	6	9	9
Square Antiprism faces are squares and equilateral triangles 	8	10	16	16
Truncated Tetrahedron formed by cutting off the corners of a tetrahedron 	12	8	18	18
Truncated Cube formed by cutting off the corners of a cube 	24	14	36	36

Using Area to Find Fraction Products

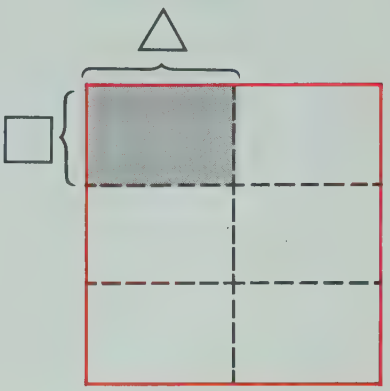
The red square is the unit of area. The length of each side of the red square is 1. Write the length (using a fraction) for the indicated segment in each □ and △. Then complete the multiplication equation.



$\frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$

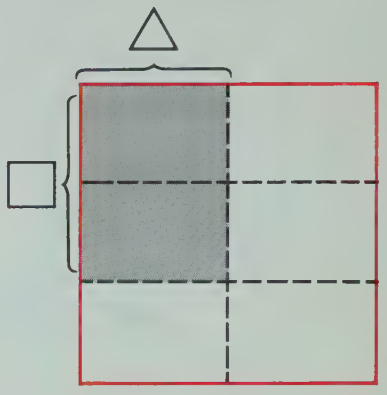
width — length — what part of the unit square is shaded?

This product is the area of the shaded part.



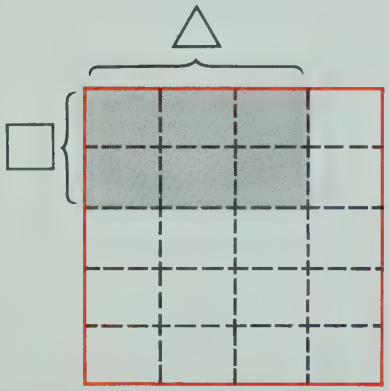
$\frac{1}{3}\square \times \triangle\frac{1}{2} = \frac{1}{6}$

area of shaded part — fraction of unit square used.

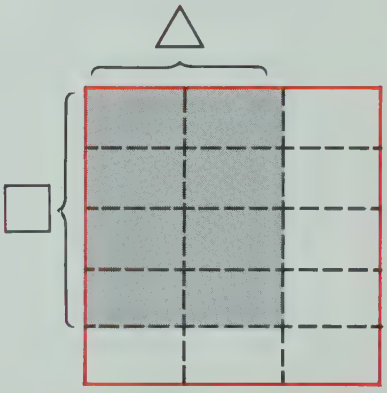


$\frac{2}{3}\square \times \triangle\frac{1}{2} = \frac{2}{6}$

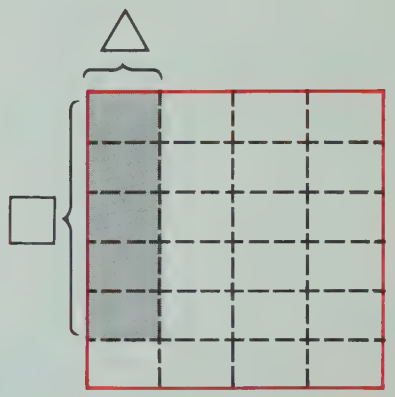
area of shaded part — fraction of unit square used.



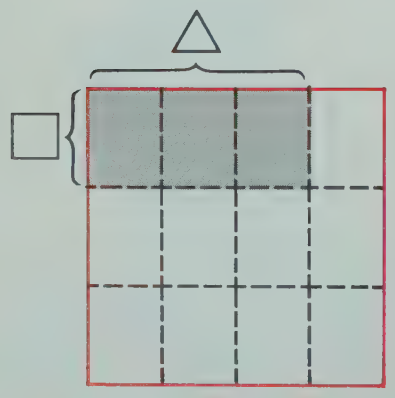
$\frac{2}{5}\square \times \triangle\frac{3}{4} = \frac{6}{20}$



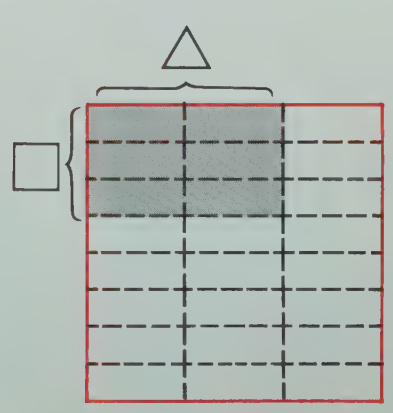
$\frac{4}{5}\square \times \triangle\frac{2}{3} = \frac{8}{15}$



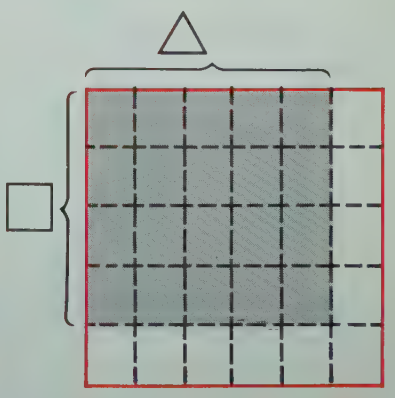
$\frac{5}{6}\square \times \triangle\frac{1}{4} = \frac{5}{24}$



$\frac{1}{3}\square \times \triangle\frac{3}{4} = \frac{3}{12}$



$\frac{3}{8}\square \times \triangle\frac{2}{3} = \frac{6}{24}$

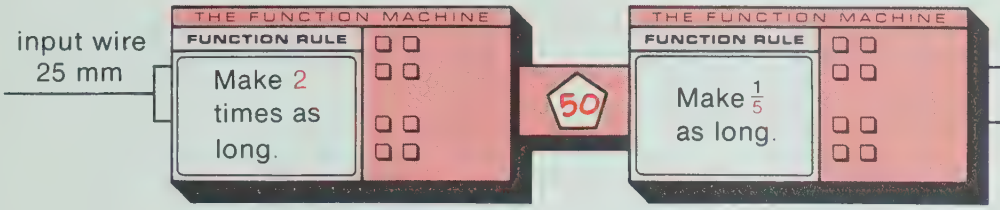


$\frac{4}{5}\square \times \triangle\frac{5}{6} = \frac{20}{30}$

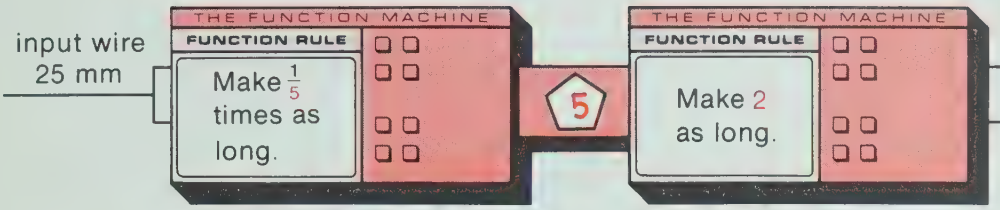
These exercises are designed to help children understand multiplication of fractions using shaded parts of regions.

Function Machines and Fraction Multiplication

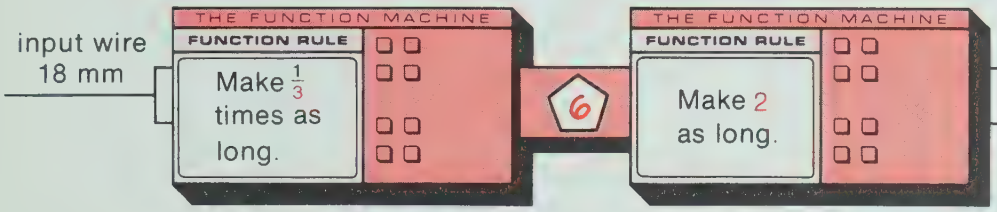
When a wire is put into the function machine it can be made longer or shorter. Give the length of the first output wire in the ⬠. Then complete the equation by giving the length of the final output wire.



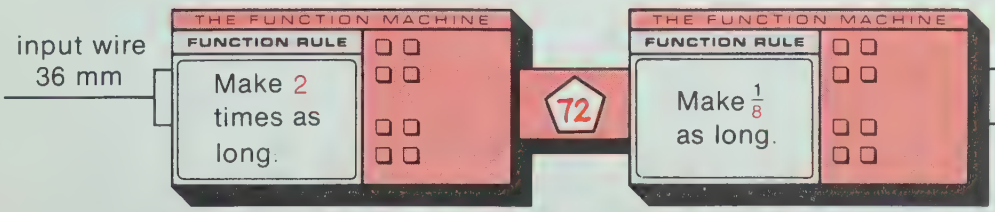
$25 \times \frac{2}{5} =$ 10



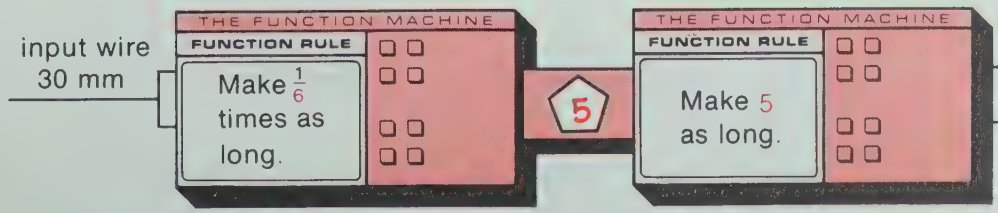
$25 \times \frac{2}{5} =$ 10



$18 \times \frac{2}{3} =$ 12



$36 \times \frac{2}{8} =$ 9



$30 \times \frac{5}{6} =$ 25

These function machine exercises are designed to help children understand the concepts of multiplying fractions with whole numbers.

Searching for Missing Factors

Find the missing factor in column 1, if possible. If not, find it in column 2.
If you can't find it in column 1 or 2, use column 3.

	Column 1		Column 2		Column 3
1.	P F F $\frac{12}{25} \div \frac{3}{5} = \frac{4}{5}$	⟷	P F F $\frac{12 \times 3}{25 \times 3} \div \frac{3}{5} =$	⟷	P F F $\frac{12 \times 3 \times 5}{25 \times 3 \times 5} \div \frac{3}{5} =$
2.	P F F $\frac{8}{21} \div \frac{2}{3} = \frac{4}{7}$	⟷	P F F $\frac{8 \times 2}{21 \times 2} \div \frac{2}{3} =$	⟷	P F F $\frac{8 \times 2 \times 3}{21 \times 2 \times 3} \div \frac{2}{3} =$
3.	$\frac{15}{24} \div \frac{5}{8} = \frac{3}{3}$	⟷	$\frac{15 \times 5}{24 \times 5} \div \frac{2}{3} =$	⟷	$\frac{15 \times 5 \times 3}{24 \times 5 \times 3} \div \frac{2}{3} =$
4.	$\frac{2}{5} \div \frac{2}{3} =$	⟷	$\frac{2 \times 3}{5 \times 3} \div \frac{2}{3} = \frac{3}{5}$	⟷	$\frac{2 \times 3 \times 2}{5 \times 3 \times 2} \div \frac{2}{3} =$
5.	$\frac{4}{7} \div \frac{2}{5} =$	⟷	$\frac{4 \times 5}{7 \times 5} \div \frac{2}{5} = \frac{10}{7}$	⟷	$\frac{4 \times 5 \times 2}{7 \times 5 \times 2} \div \frac{2}{5} =$
6.	$\frac{3}{8} \div \frac{5}{4} =$	⟷	$\frac{3 \times 5}{8 \times 5} \div \frac{5}{4} = \frac{3}{10}$	⟷	$\frac{3 \times 5 \times 4}{8 \times 5 \times 4} \div \frac{5}{4} =$
7.	$\frac{4}{7} \div \frac{3}{5} =$	⟷	$\frac{4 \times 3}{7 \times 3} \div \frac{3}{5} =$	⟷	$\frac{4 \times 3 \times 5}{7 \times 3 \times 5} \div \frac{3}{5} = \frac{20}{21}$
8.	$\frac{2}{3} \div \frac{3}{4} =$	⟷	$\frac{2 \times 3}{3 \times 3} \div \frac{3}{4} =$	⟷	$\frac{2 \times 3 \times 4}{3 \times 3 \times 4} \div \frac{3}{4} = \frac{8}{9}$
9.	$\frac{3}{8} \div \frac{4}{5} =$	⟷	$\frac{3 \times 4}{8 \times 4} \div \frac{4}{5} =$	⟷	$\frac{3 \times 4 \times 5}{8 \times 4 \times 5} \div \frac{4}{5} = \frac{15}{32}$
10.	$\frac{3}{4} \div \frac{7}{2} =$	⟷	$\frac{3 \times 2}{4 \times 2} \div \frac{7}{2} =$	⟷	$\frac{3 \times 2 \times 7}{4 \times 2 \times 7} \div \frac{7}{2} = \frac{6}{28}$
			Complete the equation to find the missing factor.		
11.	$\frac{7}{10} \div \frac{2}{3} =$	⟷	$\frac{7 \times 3}{10 \times 3} \div \frac{2}{3} =$	⟷	$\frac{7 \times 3 \times 2}{10 \times 3 \times 2} \div \frac{2}{3} = \frac{21}{20}$
			Complete the equation to find the missing factor.		

Remind the children to think of factors and products in attempting to work these exercises.

Figuring the Cost of Electrical Energy

In a recent year, the information in this table was given as the cost of operating these appliances in the "average home." This cost did not include the state and local taxes that must be added to each homeowner's electric bill.

Use the following information and figure the total cost for operating these appliances for a month (30 days).

Appliance	Cost
Electric clock	$\frac{1}{3}$¢ per day
Clothes dryer	13¢ per load
Dishwasher	3¢ per day
Refrigerator-Freezer	$7\frac{1}{2}$¢ per day
Iron	$2\frac{1}{3}$¢ per hour
Radio	$\frac{1}{5}$¢ per hour
Television	$1\frac{1}{2}$¢ per hour
Toaster	$1\frac{1}{2}$¢ for 30 slices
100-Watt light bulb	$\frac{1}{2}$¢ per hour
Clothes washer	$1\frac{1}{2}$¢ per hour

Work Space

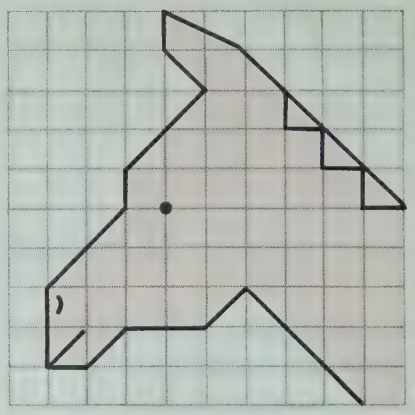
	Monthly Cost
Clock—runs all the time.	10¢
Clothes dryer—2 loads each day.	$7.80
Dishwasher—used daily.	90¢
Refrigerator-Freezer—used daily.	$2.25
Iron—used an average of 2 hours per day.	1.40
Radio—used 7 hours per day.	48¢
Television—used 10 hours per day.	4.50
Toaster—8 slices per day.	12¢
Light bulb—on 6 hours each day.	90¢
Clothes washer—used an average of 2 hours per day.	90¢
Total cost of appliances	$19.35

Inventing Graph Pictures

Follow the instructions to invent a graph picture.

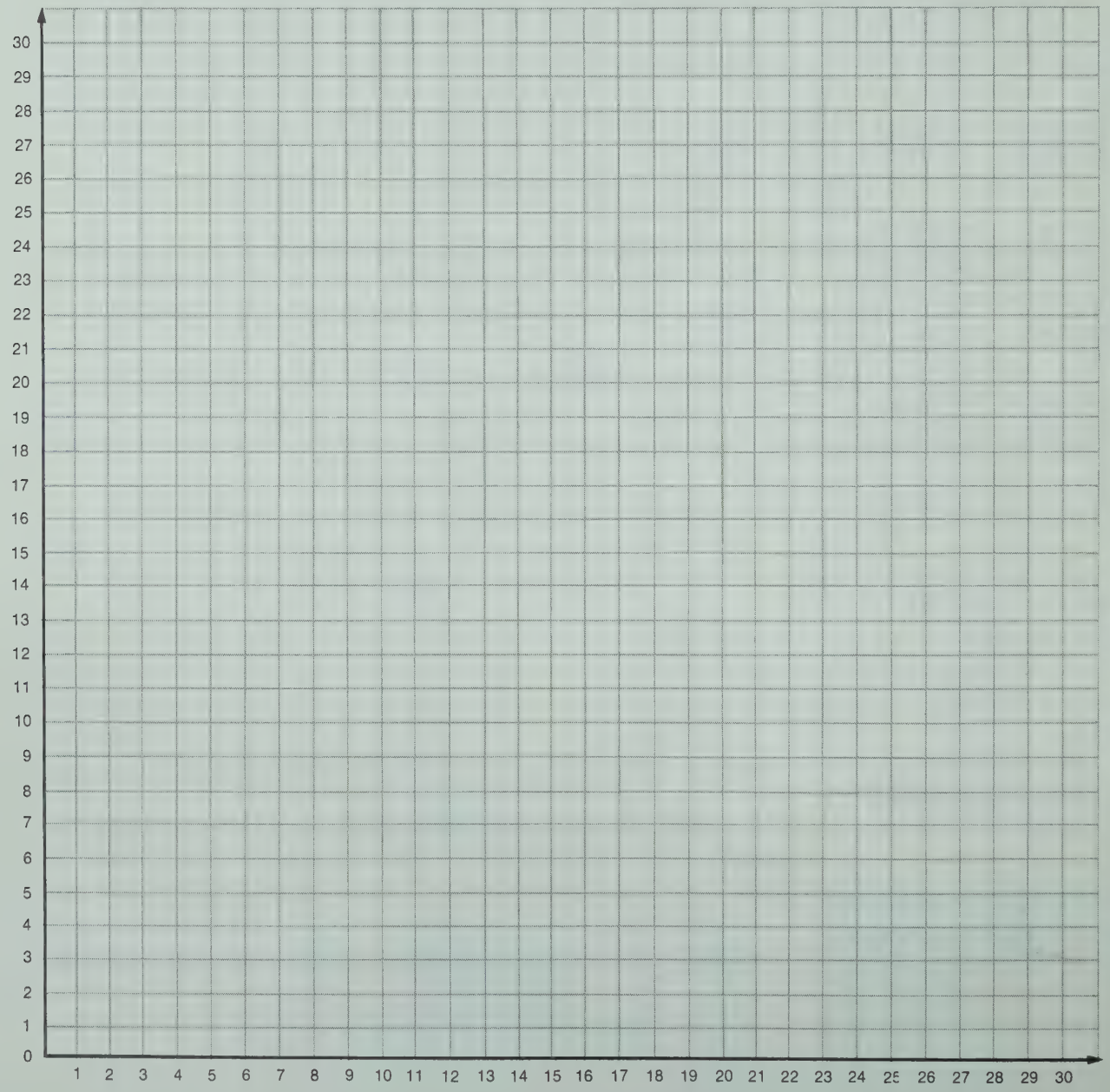
Answers will vary

1. Think of something you would like to draw.
2. Practice drawing it, using lines between points, on a spare sheet of graph paper.
3. Draw the final picture on the graph paper below.
4. List the points, in order, so someone could draw your picture by connecting the points.



Ordered points: _____

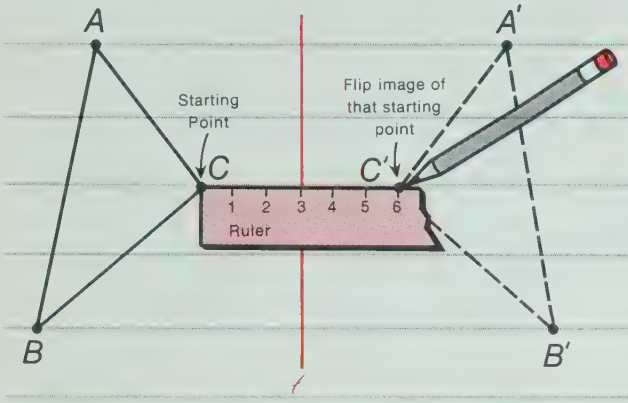
_____



Flips Without Flops

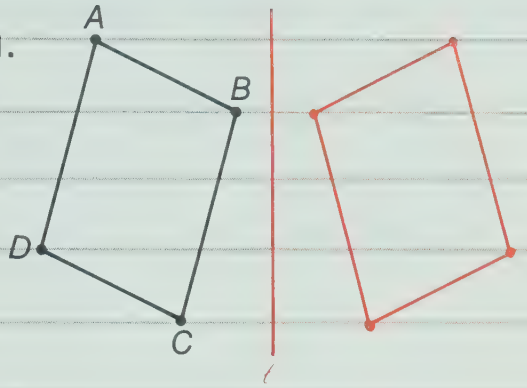
The red line in each picture is the "flip line" (line of symmetry). Study the example to find the "flip image" of any point across this line. Then use your centimeter ruler and find the "flip image" of each figure.

EXAMPLE

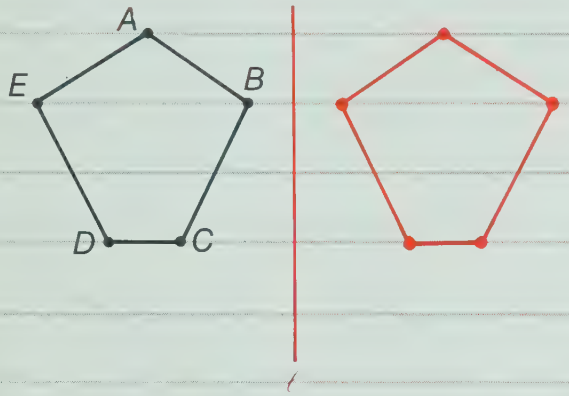


How to find the "flip image" of a figure.

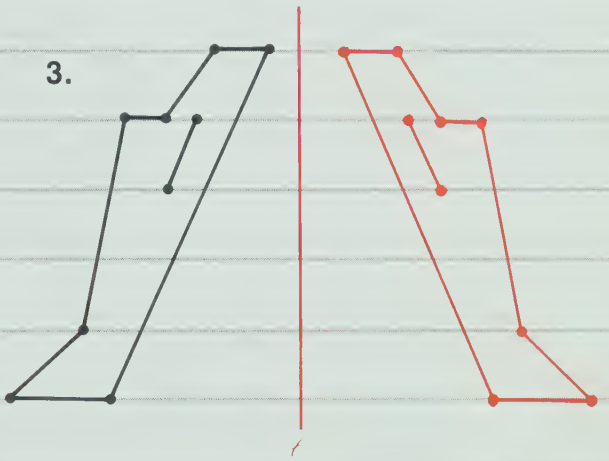
1.



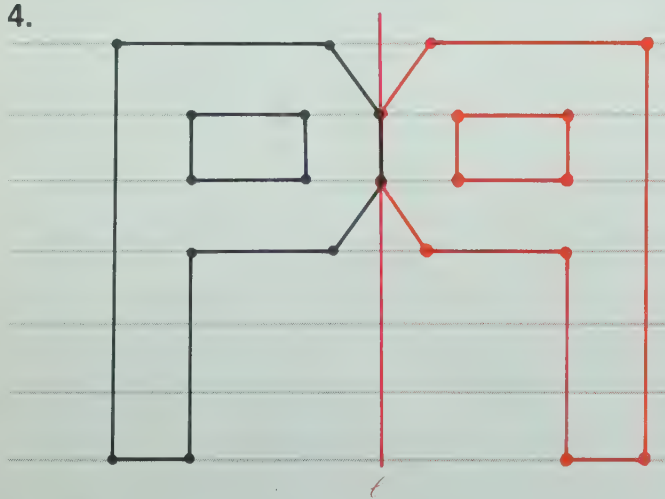
2.



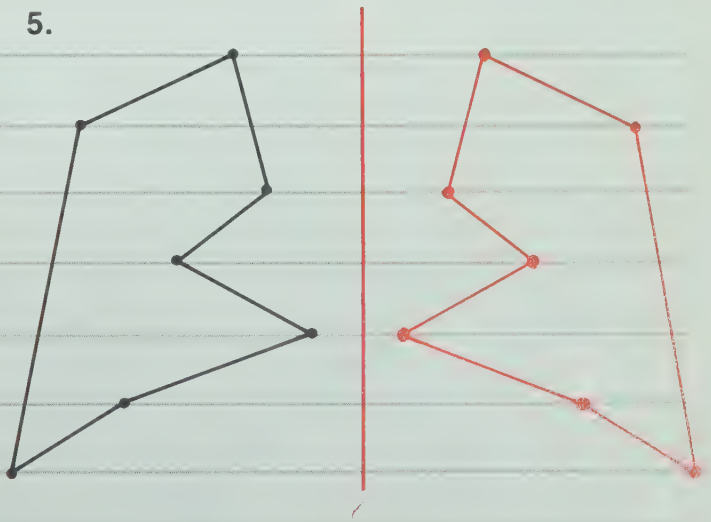
3.



4.



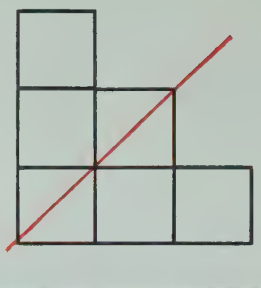
5.



As an interesting follow-up to this exercise you can have the children make cut-outs of each of the figures. Then they may check their work by flipping the figure over to see if it fits on the flip image they drew.

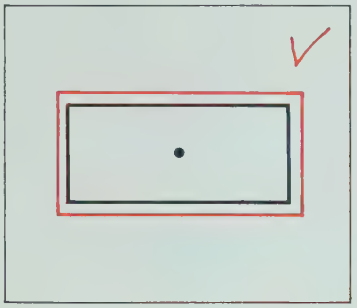
Finding Lines of Symmetry

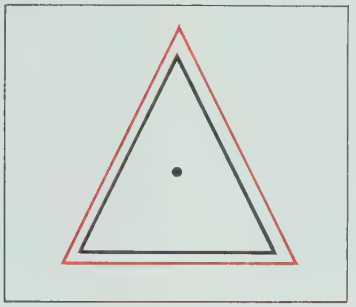
A *Hexominoe* is a pattern made by putting six squares together. (Each square must have at least one side in common with another square). This hexominoe has a line of symmetry. How many differently shaped hexominoes can you draw below which have a line(s) of symmetry? Show each line of symmetry. All possibilities shown

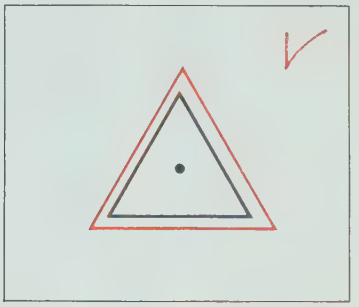


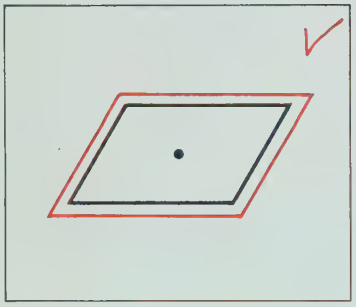
Rotating Geometric Figures

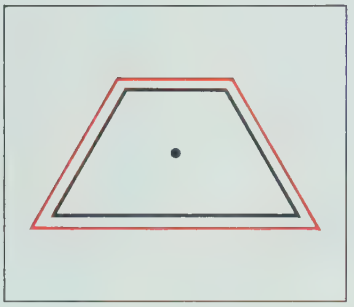
The pictures below show cardboard geometric figures fastened to a sheet of paper. A red outline is drawn around each figure. Check (✓) the figures that will fit back inside the outline when turned only part of the way around.

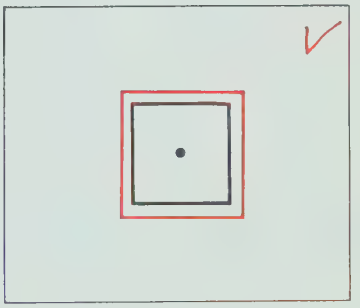
1. 

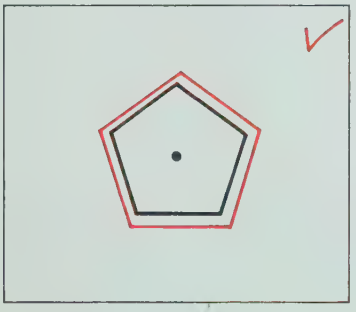
2. 

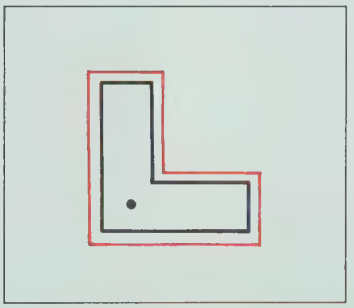
3. 

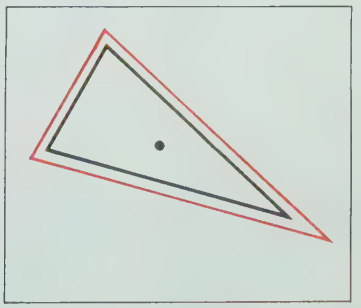
4. 

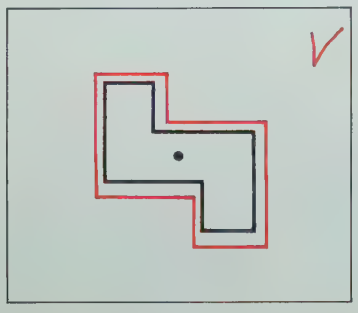
5. 

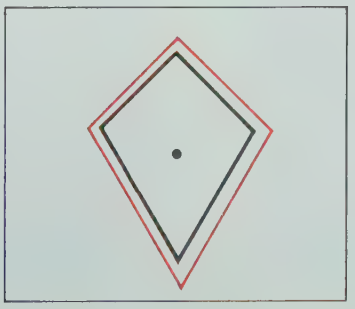
6. 

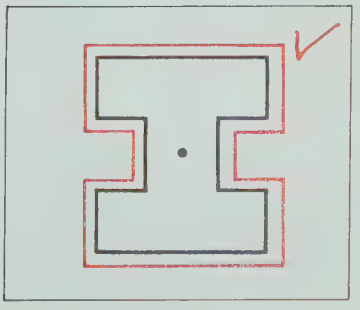
7. 

8. 

9. 

10. 

11. 

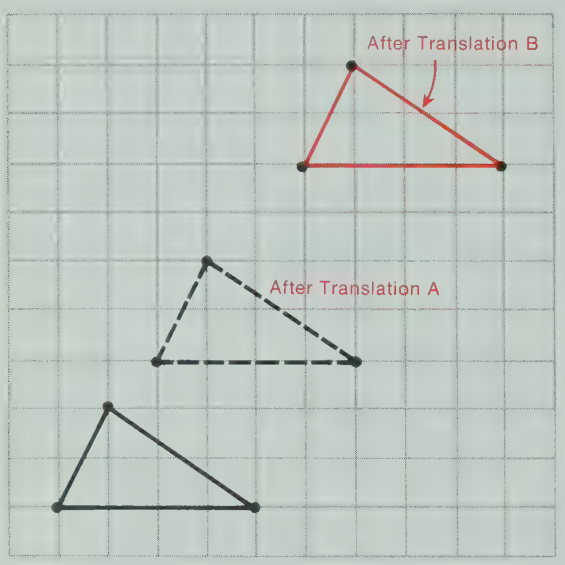
12. 

The children can check their answers to these exercises by making cut-outs of each of the figures. They may then see if the figure will fit back in after it has been rotated part of the way around the central point.

Combining Translations

Two translations (slides) are given for each part. Slide each point, using Translation A. Then slide the points using Translation B. What single slide (Translation C) would have moved the figure to the same position?

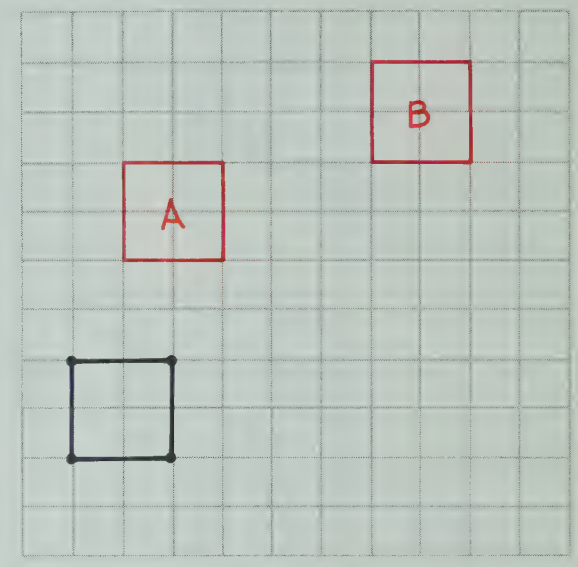
EXAMPLE:



Translation A: right 2, up 3
Translation B: right 3, up 4

Translation C: right 5, up 7

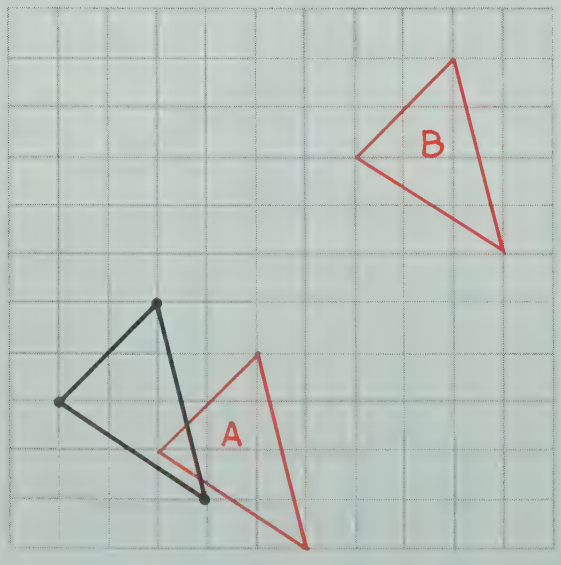
1.



Translation A: right 1, up 4
Translation B: right 5, up 2

Translation C: right 6, up 6

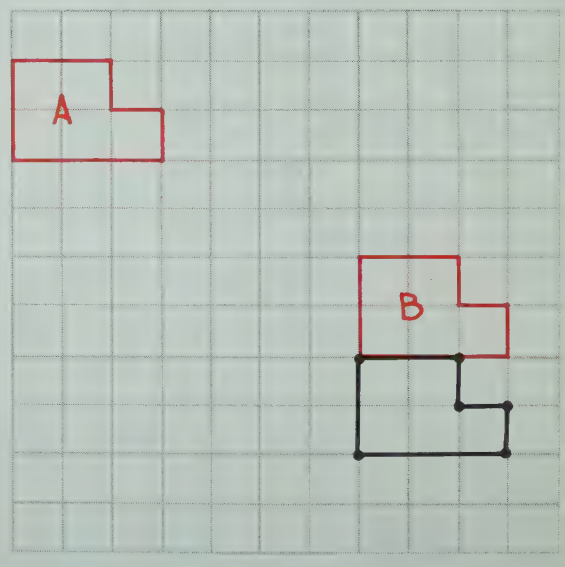
2.



Translation A: right 2, down 1
Translation B: right 4, up 6

Translation C: right 6, up 5

3.



Translation A: left 7, up 6
Translation B: right 7, down 4

Translation C: up 2

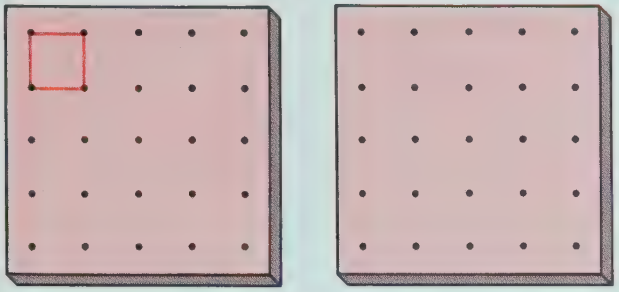
Be sure the children understand that translation C is merely the combination of A and B together.

Seeking Similar Figures

Similar figures are exactly the same shape, but do not need to be congruent. For each figure given on geoboard A, draw a figure that is similar to it on geoboard B. Choose your figure so that the two figures will not be congruent.

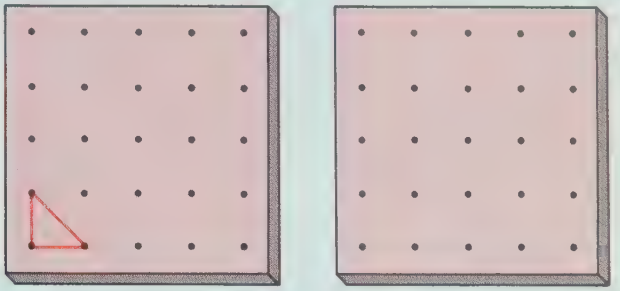
Figures may vary

1.



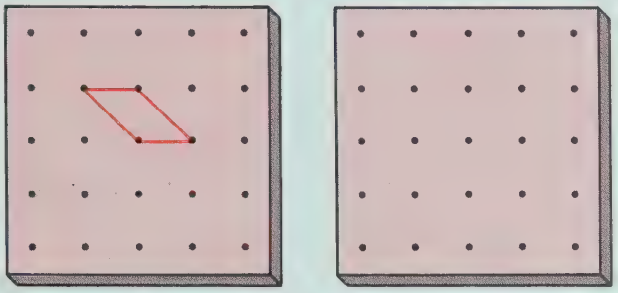
A B

2.



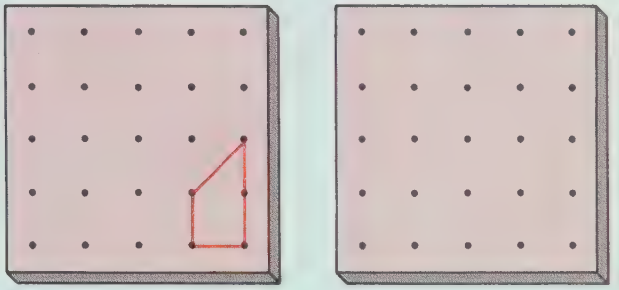
A B

3.



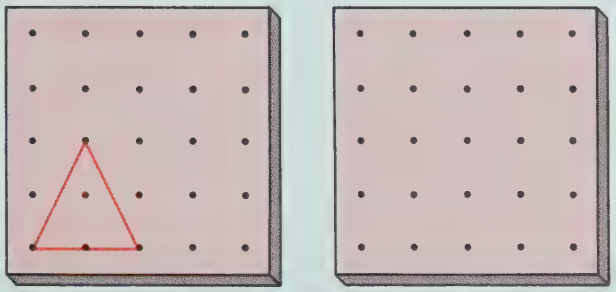
A B

4.



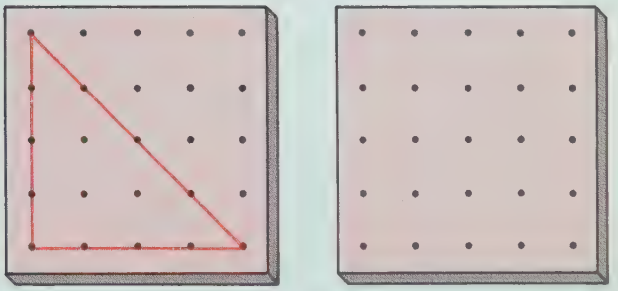
A B

5.



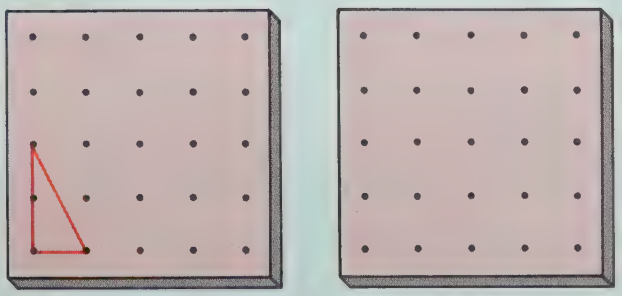
A B

6.



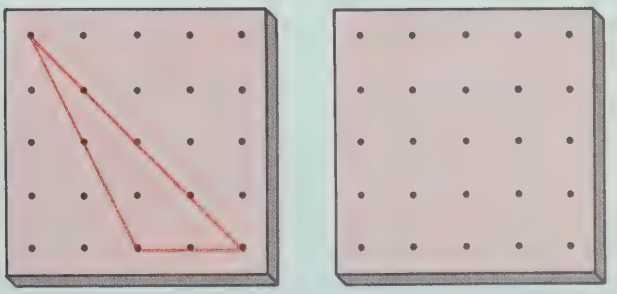
A B

7.



A B

8.

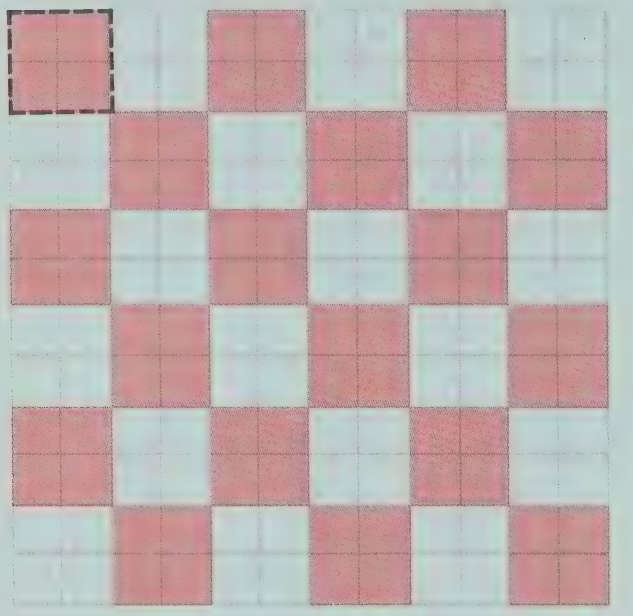


A B

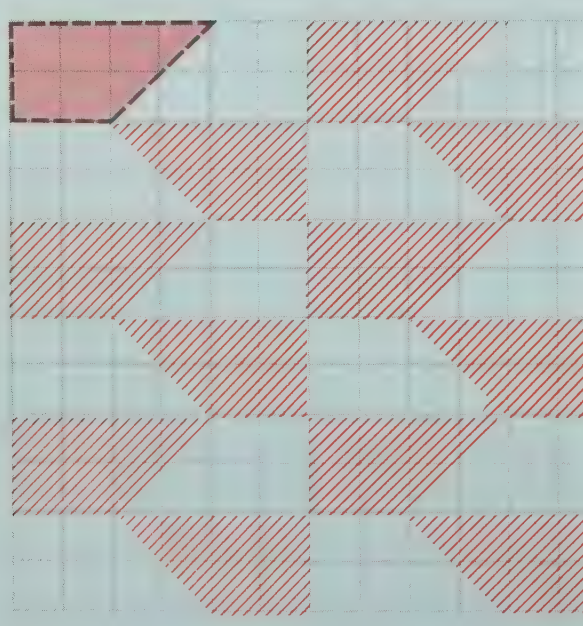
Coloring Tessellations on Square Grids

Color each grid to show the tessellation indicated. Color every other basic figure.

EXAMPLE:

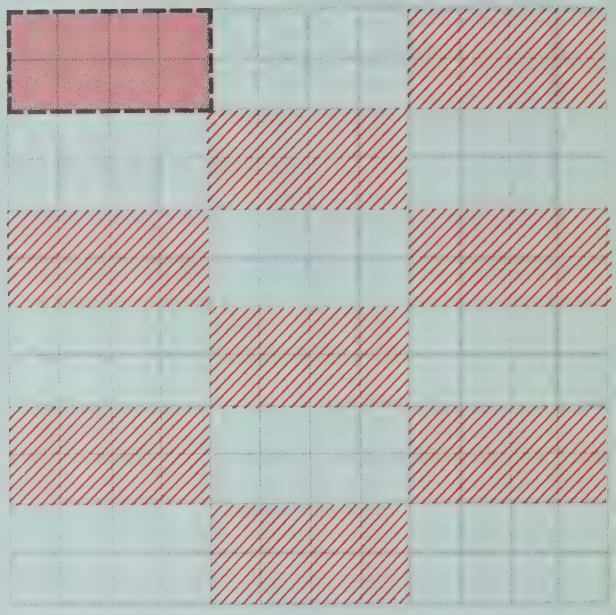


This grid is colored to show a tessellation of squares.

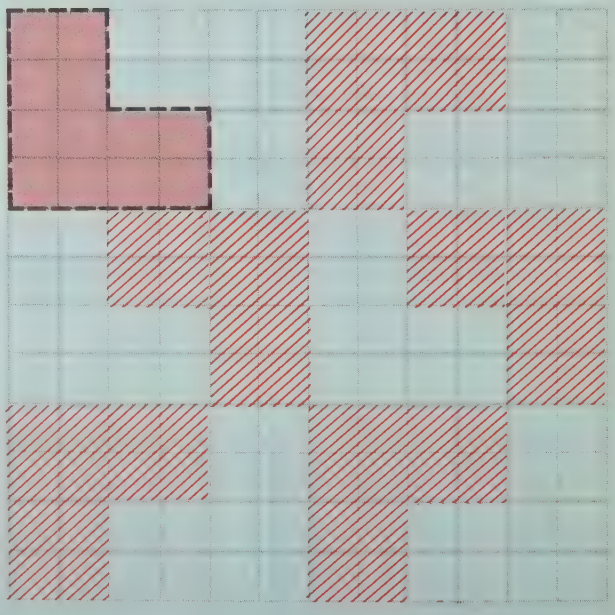


Color this grid to show a tessellation of trapezoids.

1.



Color this grid to show a tessellation of rectangles.

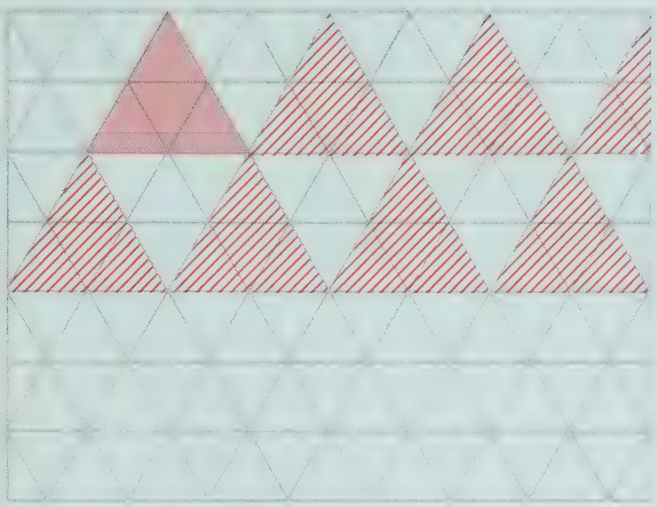


Color this grid to show a tessellation of "L" shaped hexagons.

Tessellations on Triangular Grids

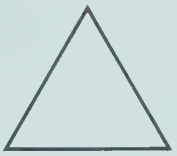
Color each grid to show the tessellation indicated. Color every other basic figure.

1.

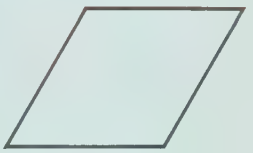


2.

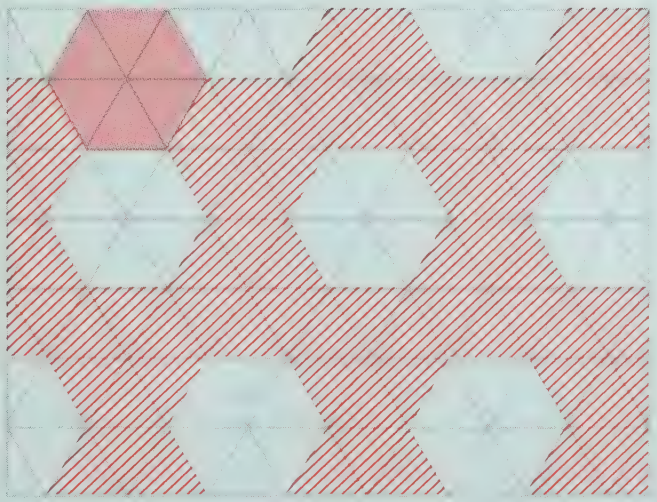
Color this grid to show a tessellation of triangles.



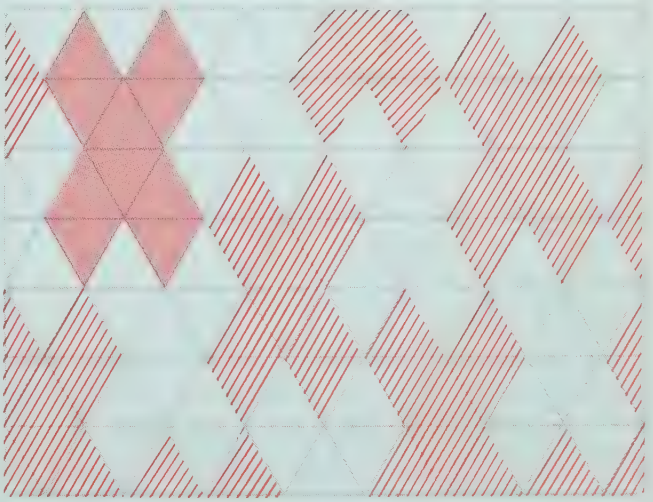
Color this grid to show a tessellation of rhombi.



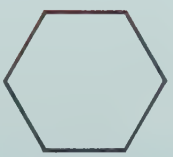
3.



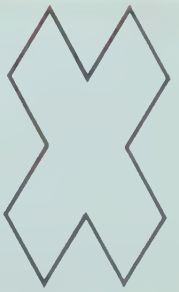
4.



Color this grid to show a tessellation of hexagons.



Color this grid to show a tessellation of "crosses."

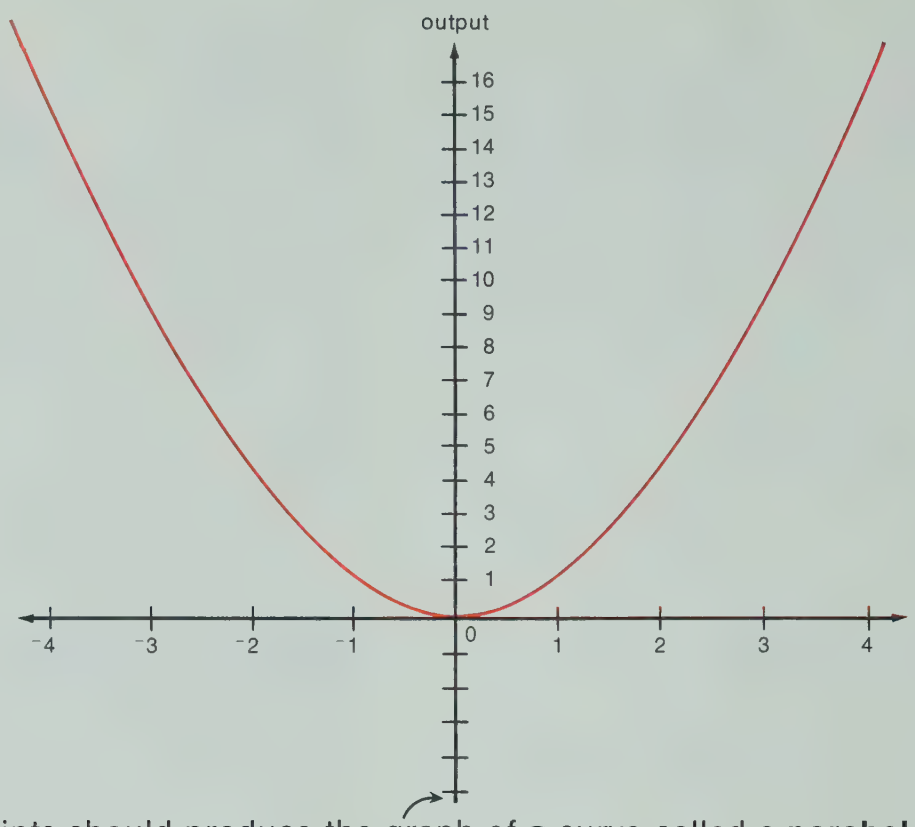


Making cut-outs of the basic figure for the tesselation may help the children understand how to complete the coloring.

Patterns in Graphs

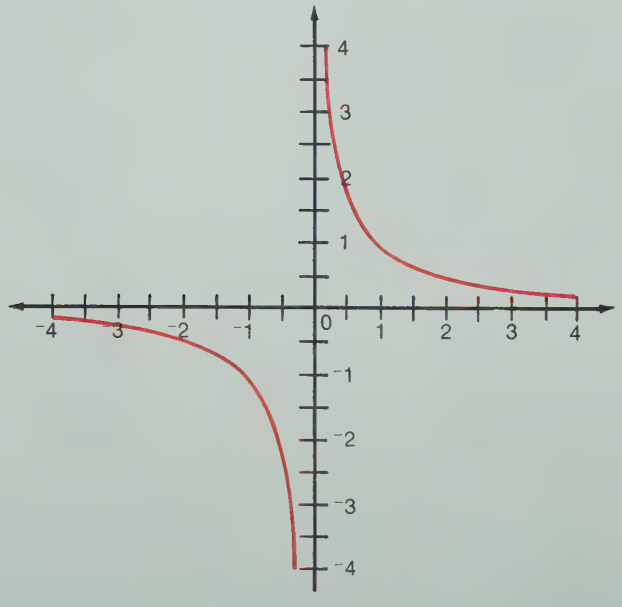
Find the pattern and complete each function table. Then graph the points and connect them with a smooth line.

input	output
4	16
3	9
2	4
1	1
0	0
$^{-}1$	1
$^{-}2$	4
$^{-}3$	9
$^{-}4$	16



These points should produce the graph of a curve called a **parabola.**

input	output
4	$\frac{1}{4}$
$^{-}4$	$-\frac{1}{4}$
3	$\frac{1}{3}$
$^{-}3$	$-\frac{1}{3}$
2	$\frac{1}{2}$
$^{-}2$	$-\frac{1}{2}$
1	1
$^{-}1$	$^{-}1$
$\frac{1}{2}$	2
$-\frac{1}{2}$	$^{-}2$
$\frac{1}{3}$	3
$-\frac{1}{3}$	$^{-}3$
$\frac{1}{4}$	4



These points should produce the graph of a curve called a **hyperbola.**

Metric Measurement	Estimating Length in the Metric System

1 **centimeter** (cm)

1 **decimeter** (dm)

A decimeter is 10 centimeters long. A **meter** (m) is 10 decimeters long.

How many centimeters long is a meter? _____

Choose a unit and estimate the following lengths. Then measure to check your estimate. Did you estimate "too high" (H), "too low" (L), or "about right" (R)?

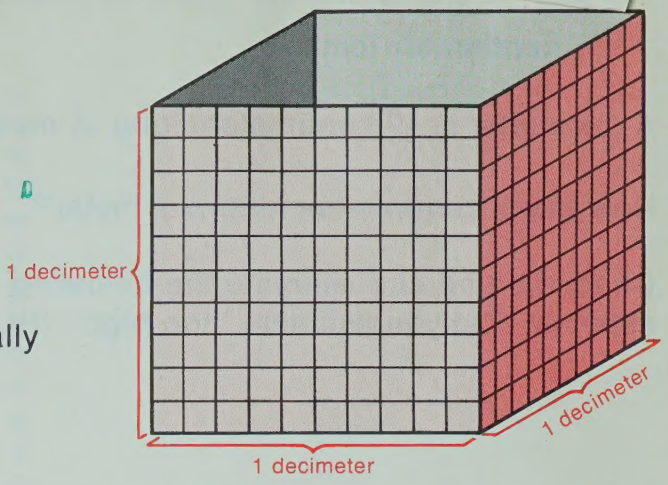
	Unit	Estimate	Actual Measure	Circle One
1. *A* — *B*	_____	_____	7cm	H L R
2. *C* — *D*	_____	_____	4cm	H L R
3. *E* — *F*	_____	_____	8.5cm	H L R
4. Width of your desk	_____	_____	_____	H L R
5. Length of your book	_____	_____	_____	H L R
6. Your height	_____	_____	_____	H L R
7. The length of your classroom	_____	_____	_____	H L R
8. Width of your hand	_____	_____	_____	H L R
9. Height of the chalkboard	_____	_____	_____	H L R
10. Width of the window	_____	_____	_____	H L R
11. Width of your classroom	_____	_____	_____	H L R
12. Length of your shoe	_____	_____	_____	H L R

Key: Tablespoon-*15 ml.*; Cup-*225 ml.*; 1 gallon-*3.84 liters* or 3840 ml.;
Juice can *177 ml.*; Large pop bottle-*480 ml.*; Sink-*(varies) 25 liters.*

Give the children considerable leeway in deciding whether or not their estimates were high, low, or about right.

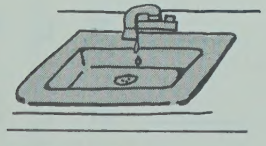
Estimating Capacity in the Metric Sy

A cubic decimeter container will hold 1 liter (ℓ) of water.



There are 1000 cubic centimeters in a cubic decimeter. A cubic centimeter container holds 1 **milliliter** (ml) of water.

Estimate the capacity in liters or milliliters of each of the following. Then find the correct capacity on the bottom of page 95. Do you usually estimate "too high" (H), "too low" (L), or "just right (R).

	Container	Unit	Estimate	Actual Capacity	Circle One
1.	 Tablespoon	ml	_____	_____	H L R
2.	 Cup	ml	_____	_____	H L R
3.	 1 Gallon container	ℓ	_____	_____	H L R
4.	 Large size pop bottle	ml	_____	_____	H L R
5.	 Kitchen sink	ℓ	_____	_____	H L R
6.	 Small frozen juice can	ml	_____	_____	H L R

Actual work with containers will provide the best experience for children completing these exercises.

EICHOLZ ■ O'DAFFER ■ FLEENOR

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